

Summary from the group exploring “Tipping Points Beyond Saddle-Node Bifurcations”. (J. Marty Anderies, Arjen Doelman, Alan Hastings, Frank Kwasniok, Giuseppe Malavolta, Clare Perryman, Jens Rademacher, Martin Rasmussen,)

- For tipping points where a parameter is moved and causes a bifurcation, if the correct indicator is identified this looks like a saddle-node bifurcation or subcritical Hopf bifurcation. e.g. the smallest Lyapounov exponent when having bifurcation on a chaotic attractor.
- Limitation in real world systems is the bifurcation indicator is not easily identifiable/ numerically cannot be found, see above example arising in ecological system of three species model (Hastings).
- Tipping may arise in multiple timescale (excitable) systems. Note however, not all systems have well defined time-scale separation.
- Tipping could be a slow (not fast) transition to another state from which it is not possible to return.
- Example: periodic orbit → saddle-node bifurcation → jump to another periodic orbit (such as no change in speed, but movement to different state).
- Real world models may need system dependent definition of tipping (e.g. for a transition to an inhospitable environment for so long it causes extinction).
- Tipping could be classed as distinct from intermittancy/flickering (where there is a deterministic or noise-induced transition between different states).
- Tipping in the non-autonomous framework: look for a bifurcation of an attractor or basin of attraction; would want to experience bifurcation in finite time not infinite time.