Introduction

During the Arctic melt season, the sea ice surface undergoes a remarkable transformation from vast expanses of snow covered ice to beautiful mosaics of ice and melt ponds. Small, disconnected ponds on the sea ice surface grow and coalesce to form much larger connected structures with complex boundaries. Melt pond area fraction can undergo a critical transition with a rapid rise from 0% to more than 70% in just a few days.

Pond fractal dimension transitions from about 1 to 2 around a critical length scale of 100 square meters in area. This type of behavior is similar to what is observed in phase transformations in statistical physics.

The transition from circles to chaotic structures in melt pond geometry can be described by the Kuramoto–Sivashinsky equation (KSE). This equation is a nonlinear PDE that has been used as a model for complex spatiotemporal dynamics in extended systems driven far from equilibrium by intrinsic instabilities.

Transition in the fractal geometry of Arctic melt ponds

- For simple objects like circles and polygons, the perimeter $P$ scales like the square root of the area $A$, that is, $P \sim \sqrt{A}$.
- However, for complex planar regions with fractal curves as their boundaries, $P \sim \sqrt[A]{D}$

where the exponent $D$ is the fractal dimension of the boundary curve.

- By analyzing area–perimeter data, Lovejoy [1] found that clouds have a fractal dimension of $D \approx 1.35$.
- Balkhanov et al. [2] calculated permafrost lakes fractal dimension in Siberia: $D \approx 1.84$.
- Hohenegger et al. [3] showed that melt ponds exhibit a transition in fractal dimension from about 1 for $A$ less than $10 \text{ m}^2$ to about 2 for $A$ greater than $1000 \text{ m}^2$.

The Kuramoto-Sivashinsky equation and melt pond pattern formation

- We apply KSE [4] that allows us to demonstrate that beginning with a critical radius the melting front of pond becomes unstable with respect to perturbations along the front.
- To describe the beginning of the fractal boundary growth, we use the linearized KSE:

$$h_1 = -a_0 h_{xx} - b_0 h_{xxx}$$

where $h$ is a normal displacement along the front; $x$ is the coordinate along the front; $x \in [0, P]$; $a_0$ and $b_0$ are positive coefficients. Since the pond boundary are closed curves, we set $P$ – periodic boundary conditions for $h$.

- Then the nontrivial solution of the linearized KSE is

$$h = C_0 \exp(ikx + \beta(k)t), \quad \beta = a_0 k^2 - b_0 k^4$$

where $k = 2\pi n / P$, $C_0$ is a constant, $m$ is a positive integer.

If $\beta(k_{\text{max}}) < 0$, then the front is stable and we can find the critical perimeter (circles) that is the same as in [3].

$$P_c = 2\pi (b_0 / a_0)^{1/2}$$

If $\beta(k_{\text{max}}) > 0$, we have an instability and $h$ increases with an exponential rate which can lead to chaotic patterns.

Conclusions

- Viewed from high above, the sea ice surface can be thought of as a two phase composite of ice and melt water. The boundaries between the two phases evolve with increasing complexity and a rapid onset of large scale connectivity, or percolation of the melt phase. The Kuramoto-Sivashinsky equation demonstrates that beginning with a critical radius the melting front becomes unstable with respect to perturbations along the front. So, pond geometry transits from circles to complex fractal structures.
- This theory can potentially provide fundamental new insights into the melting process and the evolution of sea ice albedo, as well as new definitions of tipping points through phase transitions theory.

Acknowledgment

References