Escape Rates for Tipping

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Indian Summer Monsoon

Introduction:

The Indian summer monsoon season tends to start in early June and runs through to September. The monsoon occurs due to a difference in temperature between the Indian subcontinent and the Indian Ocean. This produces winds carrying moisture as they come off the ocean onto land. The moisture is deposited over India in the form of precipitation, which then releases latent heat. Heat released, causes the land to warm even more, resulting in a greater temperature difference with the ocean and hence stronger winds. This positive feedback loop (depicted below) is the driving force of the monsoon:

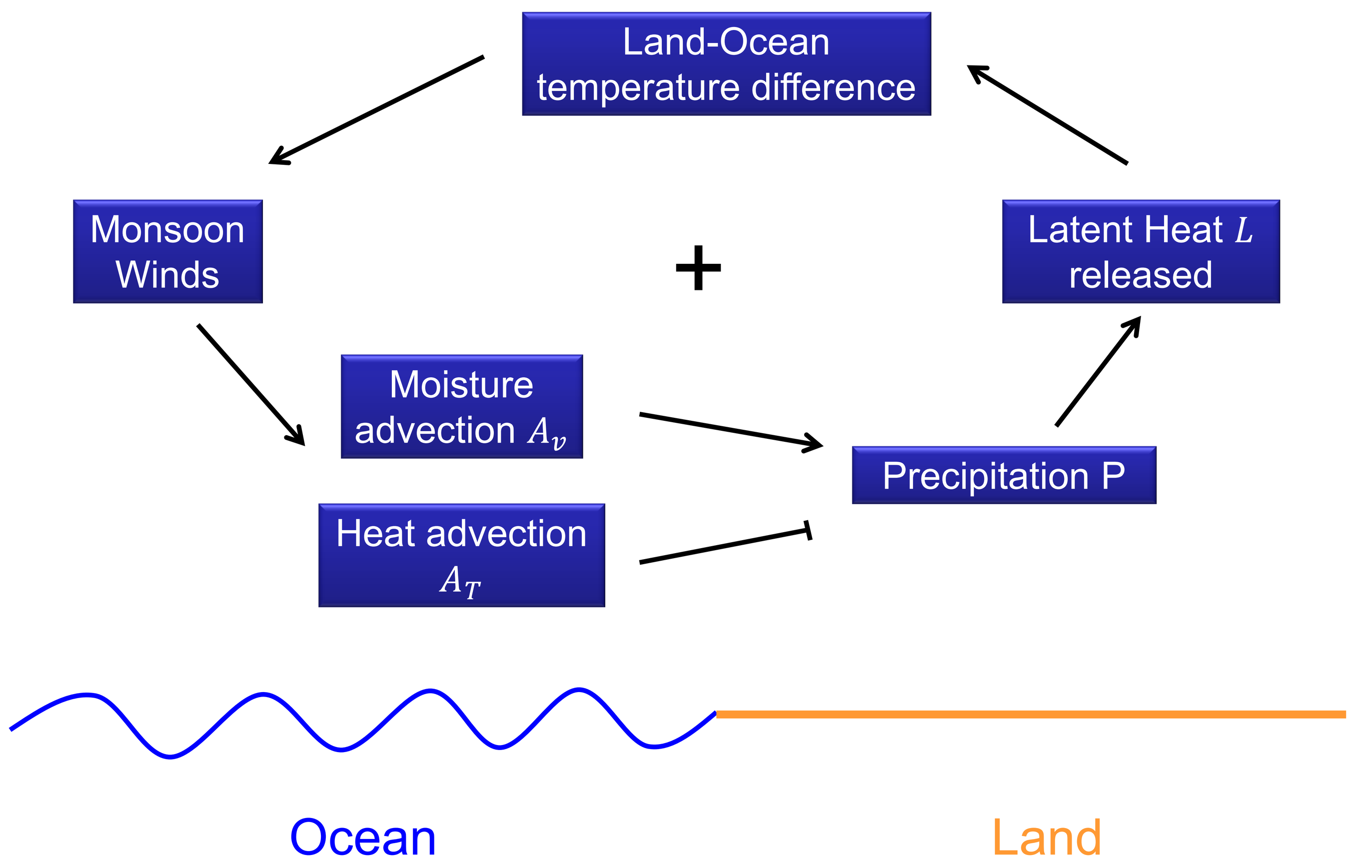


Figure: Positive feedback loop for Indian summer monsoon (based on Levermann, 2009)

Model and Methods:

For modelling the Indian summer monsoon a simplified version to that of the model presented by K. Zickfeld in her PhD thesis is used. For simplicity the model treats India as an island bounded by the Indian Ocean. This is not too much of an assumption to make considering India is surrounded by the Indian Ocean to the south, west and east and then only has the Tibetan Plateau to the north. The model includes the positive feedback mentioned above but the soil moisture feedback has been removed. This reduces Zickfeld's model from four prognostic variables to two, q_a the specific humidity and T_a the near-surface air temperature given by:

$$\dot{q}_a = \frac{E - P + A_v}{I_q}$$

$$\dot{T}_a = \frac{L(P - E) + F_{\downarrow}(1 - A_{sys}) - F_{\uparrow} + A_T}{I_T}$$

- $E(T_a, q_a)$: Evaporation
- $P(q_a)$: Precipitation
- $A_v(T_a, q_a)$: Moisture Advection
- L: Latent heat
- $F_{\downarrow}$: Incoming solar radiation
- A_{sys}: Planetary albedo
- $F_{\uparrow}(T_a)$: Outgoing long-wave radiation
- $A_T(T_a, q_a)$: Heat advection

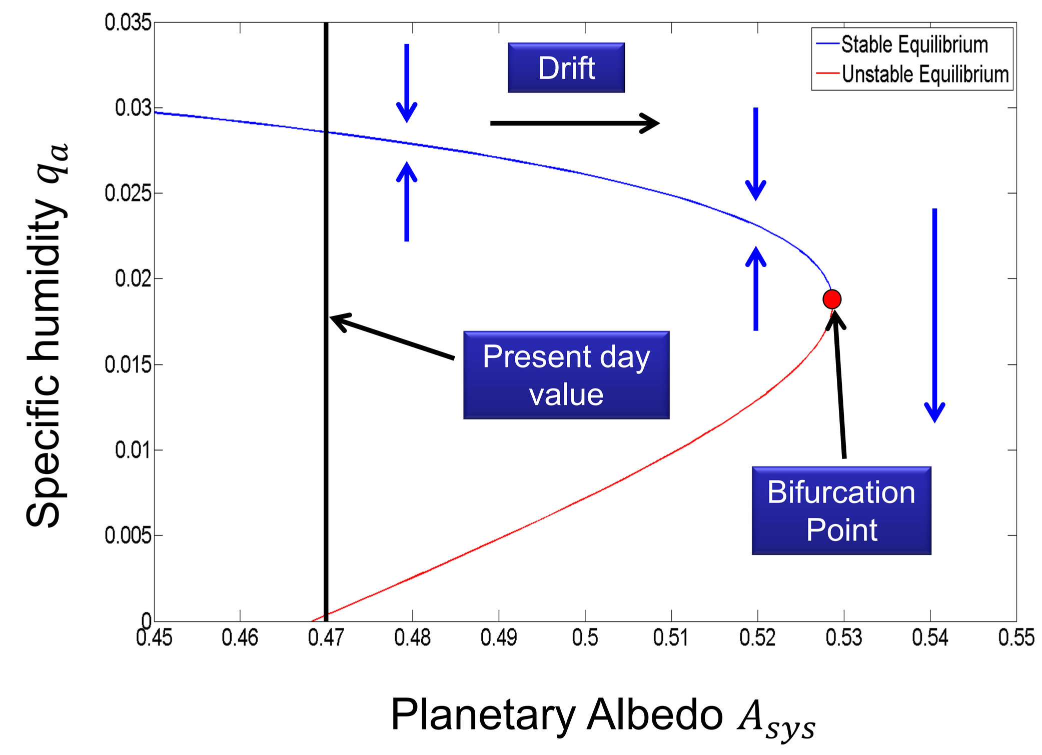
This is reduced to a 1-D model and applied to the Fokker-Planck equation with the white noise added to the planetary albedo, a driving force of the system:

$$A_{sys}(t) = A_{sys,0} + \varepsilon t + \sigma\eta$$

Drift speed (ε) — Noise level (σ)

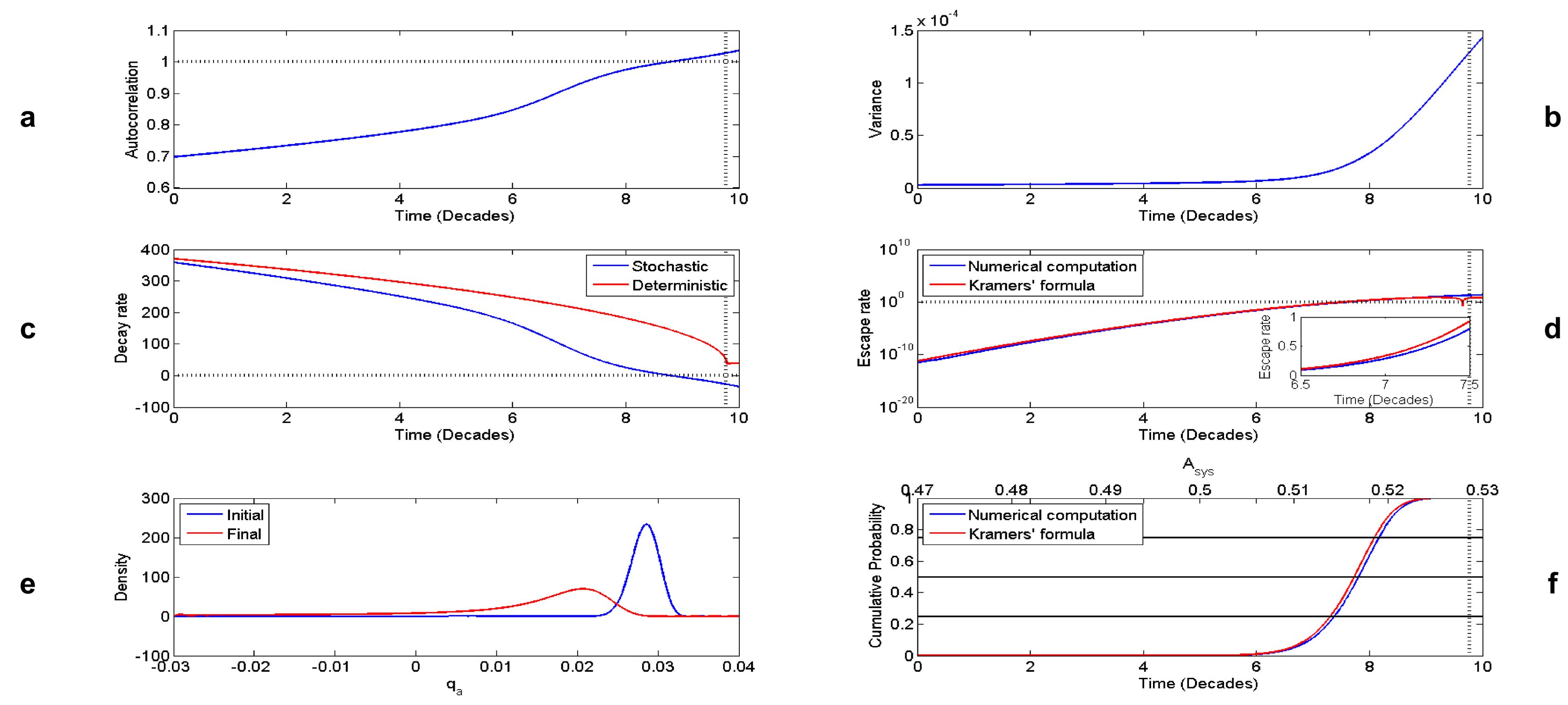
Results:

Bifurcation Diagram



It is of great interest to determine the early warning signals and escape rates by starting with the current planetary albedo and gradually drift towards the bifurcation over time.

Fokker-Planck simulations



- **Autocorrelation (a):** is the correlation between the realisations of the process per time step $\Delta t \approx 4$ days. Autocorrelation increases towards one over time.
- **Variance (b):** increases over time, both early warning indicators work in showing that a bifurcation tipping point is approached.
- **Decay rate (c):** decreases to zero (critical slowing down) as bifurcation is approached, stochastic decay rate related to autocorrelation by: $\kappa = -\frac{\log(a)}{\Delta t}$
- **Escape rate (d):** is the probability that the realisation will escape the potential well per unit time (decade).
- **Probability density function (e):** potential well flattens out over time, realisations distributed more broadly**.**
- **Cumulative probability of escape (f):** is the probability that the realisation has escaped by a given time. Once the planetary albedo reaches 0.516, there is a 50% chance that the realisation has escaped.

Future Work:

Future work will look at extending this to the 2-D Fokker-Planck equation since the time scales of specific humidity and temperature are similar. Further extensions will also look at a fast drift versus a slow drift of the system.

References:
- Levermann, A. et al. (2009), **Basic mechanism for abrupt monsoon transitions,** Proceedings of the National Academy of Sciences of the United States of America, Tipping Dynamics on Earth, Volume 106, no. 49
- Zickfeld, K. (2003), **Modeling large-scale singular climate events for integrated assessment,** Ph.D. thesis
- Zickfeld, K. et al (2005), **Is the Indian summer monsoon stable against global change?,** Geophysical Research Letters, Volume. 32

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