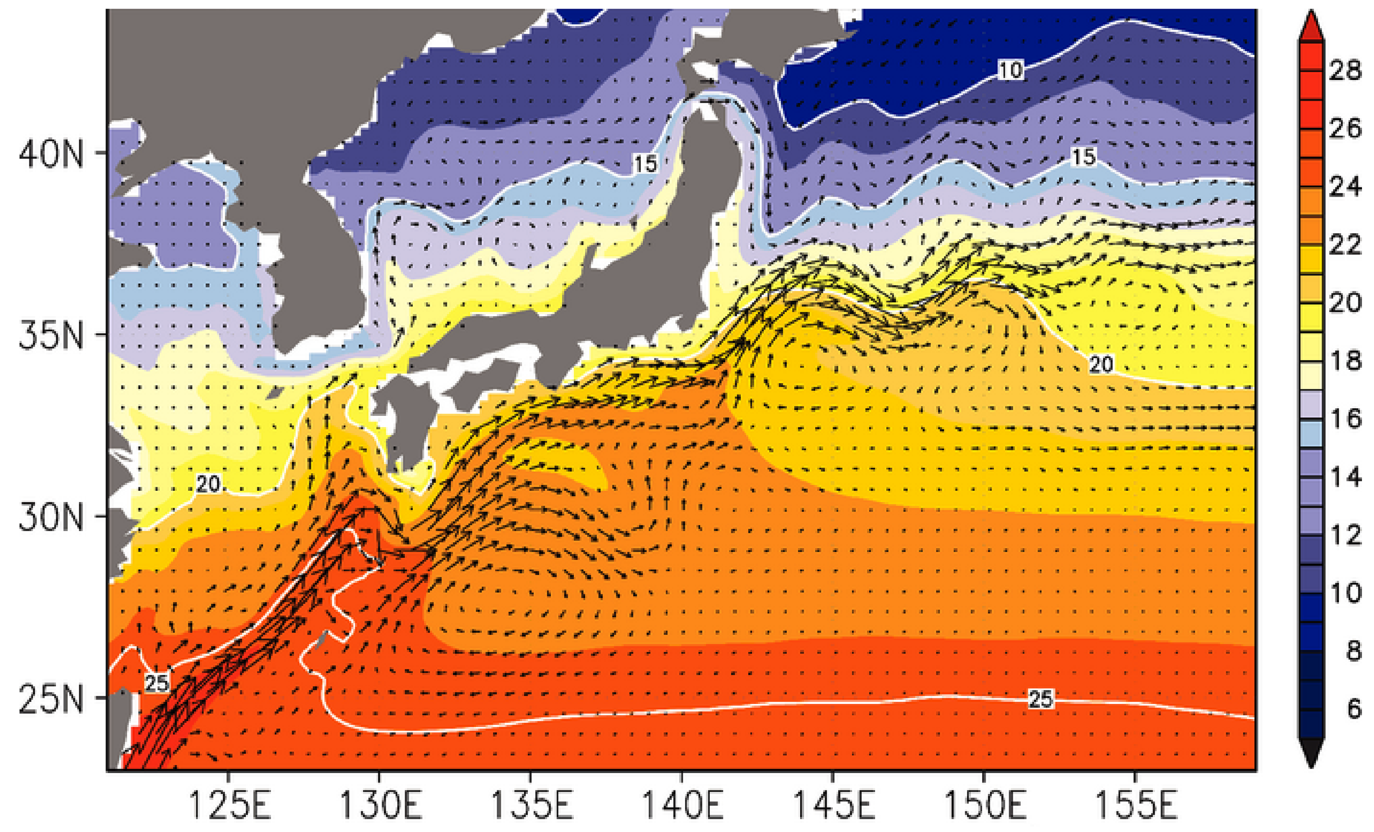
Characterizing the Kuroshio: a Random Walk to Bimodality

David Plotkin[1], Jonathan Weare[2], Dorian Abbot[1]
dplotkin@uchicago.edu
(1) University of Chicago, Department of Geophysical Sciences (2) University of Chicago, Department of Statistics

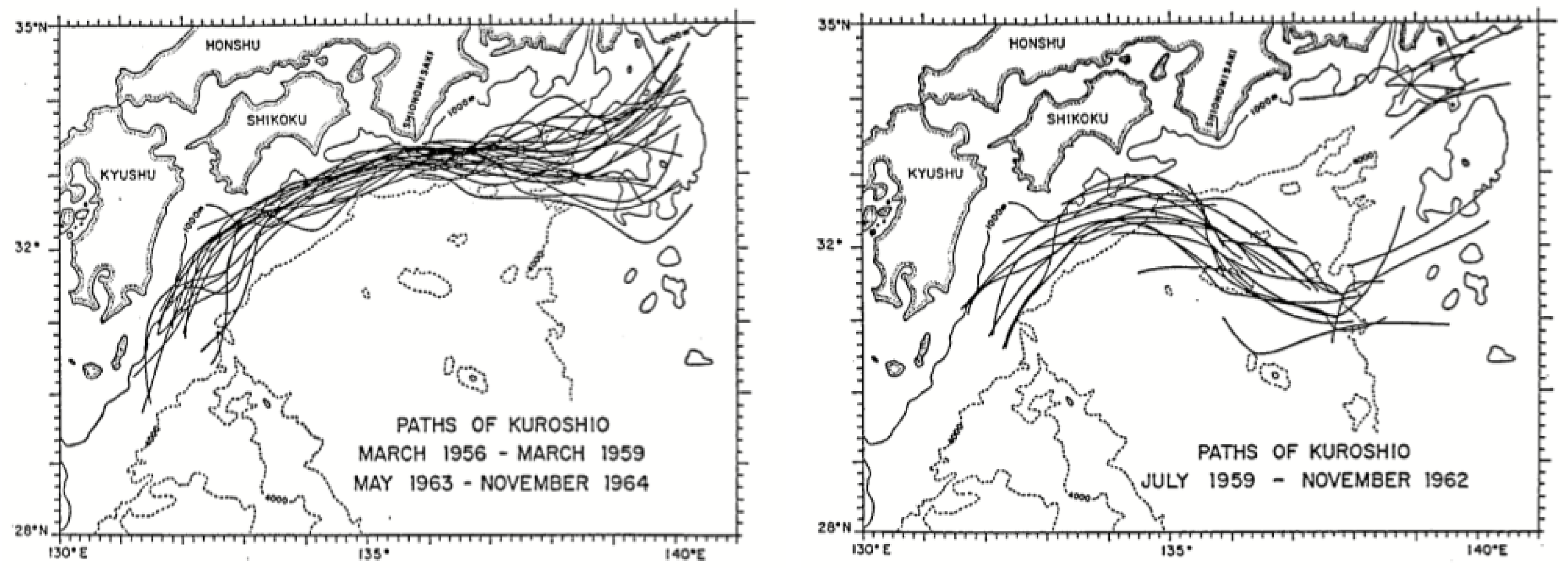
The Kuroshio

The Kuroshio western boundary current runs along the southeastern coast of Japan. It transports heat and salinity poleward, influencing large-scale weather, industry, and biology.

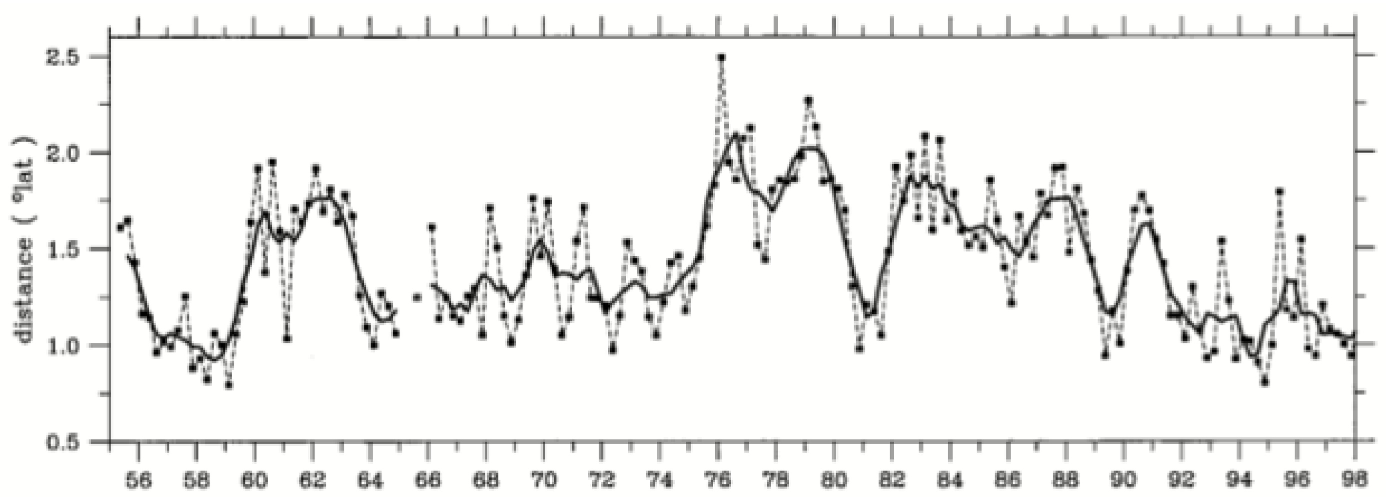


Bimodality?

The Kuroshio is widely claimed to exist in one of two persistent states: a small-meander state in which the current does not separate from the coast and a large-meander state in which the current axis is $\tilde{2}^\circ$ south of the coast at $\tilde{1}37^\circ$E. The residence time in each state is often reported as 5-10 years.



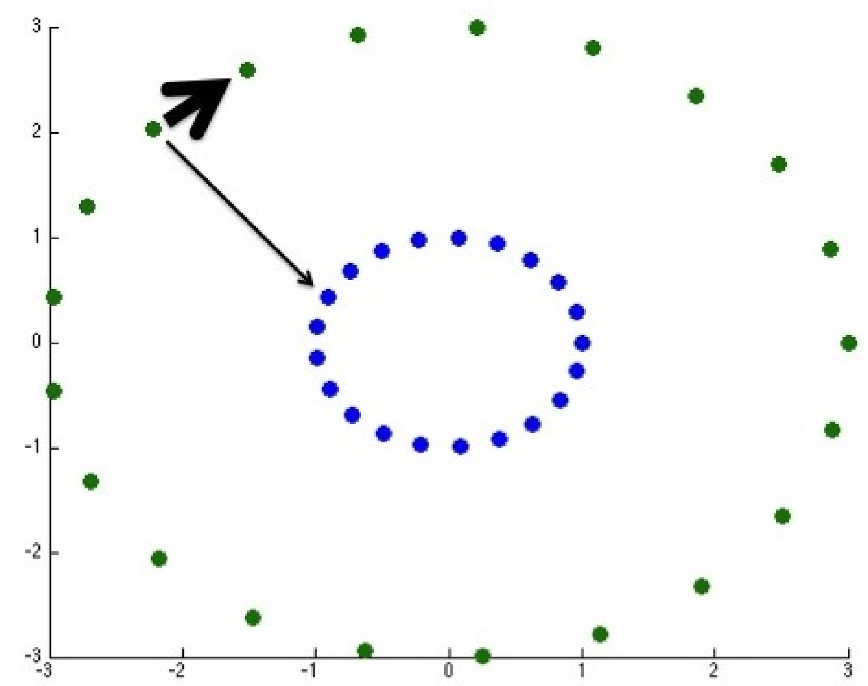
Given long residence times and relatively fast transitions between states, we expect a time series of the current axis to be bimodal, with the majority of time spent in one of the two states. Instead, indices of the axis are very noisy:



Dynamics & Non-trivial Clusters

- **The data**: a high-dimensional time series generated by a dynamical system. The data live on clusters (manifolds) in state space.
- **The problem**: standard clustering algorithms (e.g. k-means) fail if the centroids of the clusters are not spatially separated.
- **The solution**: the diffusion maps and spectral clustering algorithm treats the data as a random walk that is likely to persist on groupings of points with small pairwise separations, giving rise to clusters that would be missed by standard algorithms.

$$p_{ij} \propto e^{-\frac{|\mathbf{x_i}-\mathbf{x_j}|}{\epsilon_i \epsilon_j}} \quad (1)$$



Diffusion Maps & Spectral Clustering

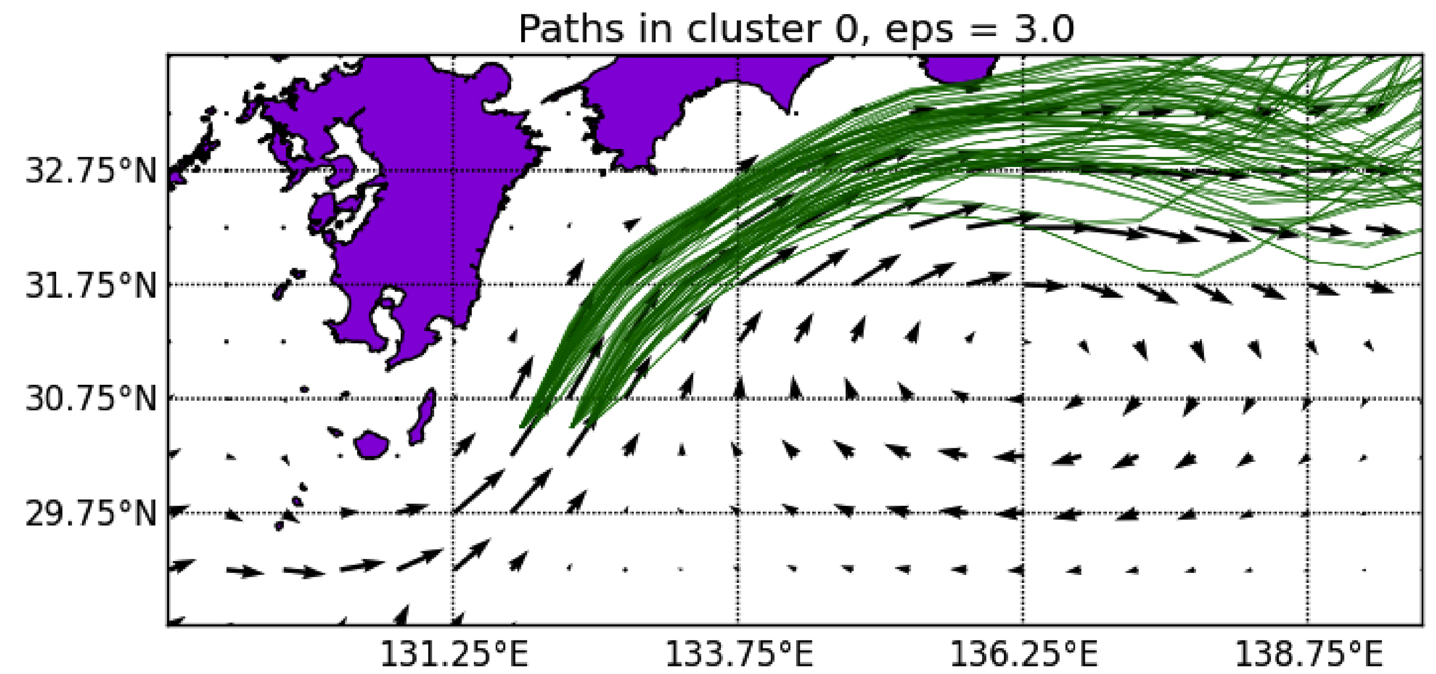
1. **Compute** the transition matrix P according to (1) using the original time series $\{\mathbf{x_i}\}_{i=1}^{N}$. Normalize so that the entries in each row and column sum to 1.
2. **Compute** the eigenvalues of P. Find a spectral gap to isolate persistent features.
3. **Construct** a low-dimesional dataset $\{\hat{\mathbf{x}}_\mathbf{i}\}_{i=1}^{N}$ using the first j eigenvectors, where j is the number of large eigenvalues preceding the spectral gap.
4. **Cluster** the low-dimesional data using a standard algorithm to obtain clusters $\{\hat{C}_i\}_{i=1}^{j}$.
5. **Map** the original time series to clusters in the high-dimensional state space using the rule:

$$\mathbf{x_i} \in C_k \iff \hat{\mathbf{x}}_\mathbf{i} \in \hat{C}_k \quad (2)$$

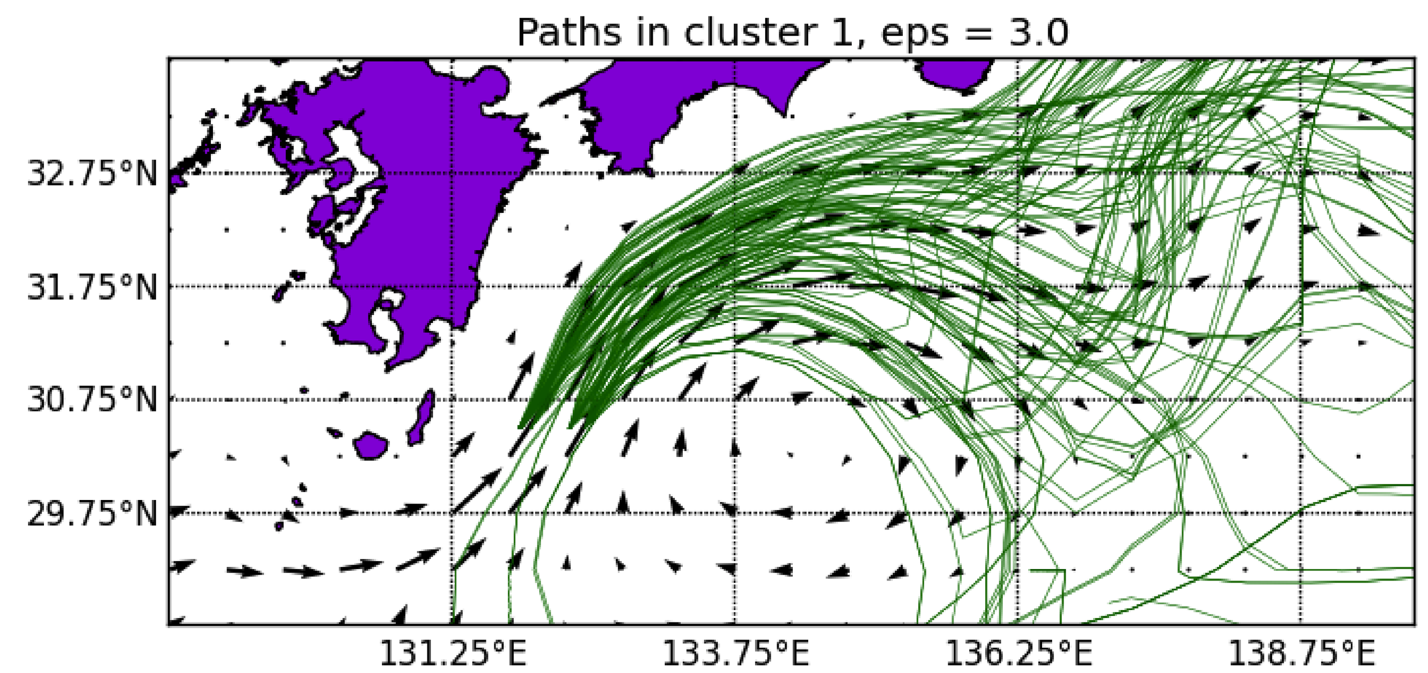
6. **Compute** the centroids of the physical clusters C_i.

Clustering the Kuroshio

- **Raw data**: Use a time series of monthly mean surface velocity fields obtained from the SODA reanalysis database.
- **For diffusion maps**: Each data point is a matrix containing a year's worth of velocity fields; done to eliminate the problem of seasonality.
- **Results**: The algorithm yields two clearly distinct states:



Paths seeded randomly in small-meander months. The background is the average velocity field in the small meander.



Paths seeded randomly in large-meander months. The background is the average velocity field in the large meander.

Conclusions

- **Mean paths are similar**: Even in the large meander, many paths stay close to the coast. This means that the mean paths in the two states are similar. This has contributed to difficulty in characterizing the states.
- **Variability is key**: The states are obviously distinct in terms of variation: small-meander paths are very similar while large-meander paths vary greatly.
- **A new index**: The location of farthest 10^{th} percentile of paths in a given month is a good measure of which meander the month falls into.
- **The recirculation gyre moves**: The center of the recirculation gyre moves about 200 km to the southwest during the large-meander regime, even when paths do not separate from the coast.