# AN ANNOTATED BIBLIOGRAPHY FOR SEA ICE STRUCTURE AND PROCESSES

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Abstract. Sea ice covers approximately 7-10% of the Earth's ocean surface, and is both an indicator and agent of climate change. The ice cover controls the exchange of heat, momentum, and gases between the ocean and atmosphere. As sea water freezes it rejects brine and increases the salinity and density of the surrounding ocean water, playing a key role in the polar halocline and the thermohaline circulation of the ocean. As a material, sea ice is a polycrystalline composite consisting of a pure ice host containing brine, air, and solid salt inclusions.

This is intended primarily as an educational resource that annotates bibliographic material related to the mathematical treatment of the structure of sea ice, its ecological role, and its role in a changing climate. The choice of material is taken, in large part, from work referenced in [2] and [12] at the Mathematical Biosciences Institute workshop 'Ocean Ecologies and Their Physical Habitats in a Changing Climate' June 2011.

## Contents

## 1. Introduction to Sea Ice

[34] D. N. Thomas and G. S. Dieckmann, editors. *Sea Ice: An Introduction to its Physics, Chemistry, Biology and Geology (2nd edition)*. Blackwell, Oxford, 2009.

This book has 15 chapters written by different contributing experts. It is an authoritative reference that provides a comprehensive view of our knowledge about sea ice.

[37] W. F. Weeks and W. D. Hibler (III). *On Sea Ice.* University of Alaska Press, 2010.

This is another excellent book on sea ice. The author (W. F. Weeks) gives a natural history of the subject, often writing in the first person. Chapter 16 on Ice Dynamics is co-authored with W. D. Hibler III. The book covers both microscopic and macroscopic properties of sea ice. The bibliography spans 52 pages. It contains several appendices that include a discussion of thin sections and remote sensing, and a glossary on sea ice terminology.

[36] W. F. Weeks and S. F. Ackley. The growth, structure and properties of sea ice. *US Army Corp of Engineers, Cold Regions Research and Engineering Laboratory, Monograph 82-1*, 1982.

This manuscript is shorter than the more recent books [34] and [37]. It describes the structural characteristics of sea ice and the relationship between these structural characteristics and its mechanical, thermal and electrical properties.

## 2. Albedo and Climate Impacts of Sea Ice

[29] D. K. Perovich, J. A. Richter-Menge, K. F. Jones, and B. Light. Sunlight, water, and ice: Extreme Arctic sea ice melt during the summer of 2007. *Geophys. Res. Lett.*, 35, 2008.

Open ocean reflects about 7% of the incident solar radiation, compared to 65% and 85% reflection by bare sea ice and snow covered ice, respectively. The highly reflecting ice is replaced by highly absorbing ocean as the ice cover decays, resulting in more radiation absorption and more melting. This is the so called ‘ice-albedo feedback mechanism.’

Observations made with ice mass balance buoys that drifted with the ice pack in the Beaufort Sea and North Pole regions of the Arctic indicate a massive amount of bottom melting of the ice in the Beaufort Sea during the summer of 2007. The authors summarize the data and present arguments that the bottom melting was a result of solar heating of the upper ocean. (In 2008 when this was published, the minimum extent of the Arctic sea ice cover ever recorded had just occurred, in September 2007. A new record low has now been set in September 2012 (as of 2012) [28].)

[9] I. Eisenman and J. S. Wettlaufer. Nonlinear threshold behavior during the loss of Arctic sea ice. *Proc. Natl. Acad. Sci. USA*, 106:28–32, 2009.

The authors consider the ice-albedo feedback mechanism and its role in the existence of future Arctic sea ice. This is done by using a bifurcation analysis of an energy balance model for the surface of the Arctic Ocean. Arguments are presented for how Arctic sea ice may reach an ice free state if the climate passes through a ‘tipping point’.

[1] D. S. Abbot, M. Silber, and R. T. Pierrehumbert. Bifurcations leading to summer Arctic sea ice loss. *J. Geophys. Res.*, 116:D19120, 2011.

This extends [9] by including a cloud cover into energy balance models. In short, the article addresses the impact clouds have on the Arctic sea ice cover. Different 'tipping points' are observed which change depending on whether or not clouds have a protective or destructive feedback effect.

[35] S. Tietsche, D. Notz, J. H. Jungclaus, and J. Marotzke. Recovery mechanisms of Arctic summer sea ice. *Geophysical Research Letters*, 38:2707, January 2011.

In contrast to the analysis in [1] and [9], the authors conclude that a tipping point during the decline of the Arctic sea ice in the 21st century is unlikely to occur. The authors examine the recovery of Arctic sea ice from a prescribed ice-free condition by running simulations with an Atmosphere-Ocean General Circulation Model (AOGCM), pointing to the value of such experiments that systematically perturb Arctic sea ice conditions. Their results suggest the existence of large scale recovery mechanisms that alleviate the destructive ice-albedo feedback.

Their model incorporates a dynamic thermodynamic sea ice model based on the work of Hibler [21], which contains much of the underlying mathematical analysis.

## 3. Fluid Transport in Sea Ice

[14] K. M. Golden, S. F. Ackley, and V. I. Lytle. The percolation phase transition in sea ice. *Science*, 282:2238–2241, 1998.

Percolation theory is used to explain the phase transition that occurs in the fluid transport properties of columnar sea ice at a 5% brine volume fraction. Columnar sea ice is effectively impermeable to fluid flow below a 5% brine volume fraction, yet is permeable for brine volume fractions above this threshold value. This 'on-off' switch has become known as the 'rule of fives' and is commonly used in research presented in many subsequent publications and investigations. This is the first major result that evaluates sea ice using a percolation theory approach commonly used when investigating composite materials.

[18] K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and J. Zhu. Thermal evolution of permeability and microstructure in sea ice. *Geophys. Res. Lett.*, 34:L16501, 2007.

The article assesses the fluid permeability of sea ice using X-ray computed tomographic images, theoretical bounds, percolation theory, and hierarchical models. These are used to accurately describe *in situ* permeability field data. The article addresses the impact the brine volume fraction and microstructure have on the effective fluid transport properties of sea ice.

[30] D. J. Pringle, J. E. Miner, H. Eicken, and K. M. Golden. Pore-space percolation in sea ice single crystals. *J. Geophys. Res. (Oceans)*, 114:C12017, 2009.

This paper examines the fluid permeability of single crystals of sea ice, using X-ray computed tomographic images and percolation theory. The authors observe a notable difference between the vertical and horizontal permeabilities in single crystals of sea ice and present an argument based on sea ice anisotropy to account for the difference. Further, a modified brine volume threshold value in traditional vertical columnar sea ice percolation models allows for accurate modeling of the observed horizontal permeabilities.

## 4. Snow-Ice Formation

[3] S. F. Ackley, V. I. Lytle, K. M. Golden, M. N. Darling, and G. A. Kuehn. Sea ice measurements during ANZFLUX. *Antarctic J. U. S.*, 30:133–135, 1995.

Snow-ice formation is presented as a critical element for sea ice growth in the Antarctic. During the 1994 Antarctic ANZFLUX expedition, a large heat flux from the ocean was observed, and the sea ice pack should have melted within a month. When this melting was not observed, it was postulated that the observed snow-ice formation was allowing the ice to grow at a rate similar to which it was melting. Thus, the sea ice thickness remained relatively unchanged.

[24] T. Maksym and M. O. Jeffries. A one-dimensional percolation model of flooding and snow ice formation on Antarctic sea ice. *J. Geophys. Res.*, 105(C11):26,313–26,331, 2000.

This article examines snow-ice formation and provides a rigorous and detailed description of the process. Two one-dimensional models are presented. The first ‘simple model’ primarily investigates the effect of snow loading on the surface of the ice, while the second ‘standard model’ further includes the effects of brine percolating through the sea ice (using the ‘rule of fives’ as described in [14]). A comparison with data sets is made. The analysis accounts for the many factors that drive snow-ice formation.

[25] T. Maksym and T. Markus. Antarctic sea ice thickness and snow-to-ice conversion from atmospheric reanalysis and passive microwave snow depth. *J. Geophys. Res.*, 113:C02S12, 2008.

While [24] investigates snow-ice formation on a microstructural scale, this publication focuses on the macroscopic aspects of snow-ice from the perspective of the entire Antarctic ice pack. Snow fall data and snow accumulation data are first considered. Next, the amount of snow-ice created in different regions is interpolated, and from that overall sea ice thicknesses are estimated. In particular, a process using ICESat (Ice, Cloud, and land Elevation Satellite) to determine ice thicknesses is described. The article provides a good example of a way to ‘parametrize’ a microscale process into a larger-scale system.

## 5. Thermal Transport

[23] V. I. Lytle and S. F. Ackley. Heat flux through sea ice in the Western Weddell Sea: Convective and conductive transfer processes. *J. Geophys. Res.*, 101(C4):8853–8868, 1996.

The authors present a detailed experimental analysis of thermal transport in sea ice. Observations at several experimental sites revealed the existence of convective fluid transport processes that occur when the ice is porous, thus accelerating the heat transport. This is a well written article that provides a good foundation for understanding heat transport in sea ice.

## 6. Sea Ice Microbiology

[33] D. N. Thomas and G. S. Dieckmann. Antarctic sea ice–a habitat for extremophiles. *Science*, 295:641–644, 2002.

This concise article examines the microbiology of sea ice. It focuses on the important survival mechanisms that organisms have developed to survive in the extreme conditions of sea ice. These include adaptations to survive cold temperatures, high salinities, and low light. It is also briefly mentioned that knowledge about such extremophiles may be useful when trying to detect life on other planets.

[11] C. H. Fritsen, V. I. Lytle, S. F. Ackley, and C. W. Sullivan. Autumn bloom of antarctic pack-ice algae. *Science*, 266(5186):782–784, 4 November 1994.

The biological impact of the 'rule of fives' (see [14] in §3) is considered. An algae bloom observed in the upper layers of a sea ice column displayed rapid growth while the ice was porous to fluid transport. As the temperature dropped, it was observed that the brine volume fraction dropped below the critical 5% in the layer hosting the algae. Thus, the brine pockets became isolated, and the algae quickly depleted the available nutrients.

[22] C. Krembs, H. Eicken, and J. W. Deming. From the Cover: Exopolymer alteration of physical properties of sea ice and implications for ice habitability and biogeochemistry in a warmer Arctic. *Proceedings of the National Academy of Science*, 108:3653–3658, March 2011.

The authors examine the effects that algal extracellular polymeric substances (EPS) have on sea ice microstructure and its physical properties. A series of experiments are presented, and the observed differences in bulk salinity and ice permeability between ice with EPS and without EPS are discussed. The broader impacts of EPS in sea ice as the climate warms are also briefly mentioned, and the introduction provides references addressing the state of algae growth in the Arctic as summer sea ice diminishes. In contrast to [11], which examines the effects that sea ice microstructures have on biological activities, this article examines the effects that biological activities have on sea ice microstructure.

## 7. Sea Ice Permittivity and Remote Sensing

[4] S. A. Arcone, A. J Gow, and S. McGrew. Structure and dielectric properties at 4.8 and 9.5 GHz of saline ice. *J. Geophys. Res.*, 91(C12):14281–14303, 1986.

Complex permittivity measurements of sea ice taken at 4.8 and 9.5 GHz are evaluated. The article describes several years of precise experiments that include crystallographic information, temperatures, and salinities. The authors provide a rigorous experimental approach and the analysis has laid groundwork for subsequent research.

[16] K. M. Golden, M. Cheney, K. H. Ding, A. K. Fung, T. C. Grenfell, D. Isaacson, J. A. Kong, S. V. Nghiem, J. Sylvester, and D. P. Winebrenner. Forward electromagnetic scattering models for sea ice. *IEEE Trans. Geosci. Rem. Sens.*, 36(5):1655–1674, 1998.

This provides an analysis of forward electromagnetic scattering models for sea ice, focusing on the complex permittivity of sea ice and remote sensing. Several theoretical forward bounds that are closely associated with the work of the first author are compared with sea ice data.

[15] K. M. Golden, D. Borup, M. Cheney, E. Cherkaev, M. S. Dawson, K. H. Ding, A. K. Fung, D. Isaacson, S. A. Johnson, , A. K. Jordan, J. A. Kong, R. Kwok, S. V. Nghiem, R. G. Onstott, J. Sylvester, D. P. Winebrenner, and I. Zabel. Inverse electromagnetic scattering models for sea ice. *IEEE Trans. Geosci. Rem. Sens.*, 36(5):1675–1704, 1998.

This sequel to [16] considers inverse electromagnetic scattering models for sea ice. The objective of the inverse problem is to recover microstructural characteristics of sea ice and other characteristics of interest from remote electromagnetic sensing data. Examples of microstructural characteristics are: brine and air volume factions, brine and air inclusion size, connectedness properties of the brine phase, and isotropy or anisotropy of brine microstructure. Other characteristics of interest are the thickness of *thin ice*, which has a major influence on the winter heat transfer from the ocean to the atmosphere.

[13] K. Golden and G. Papanicolaou. Bounds for effective parameters of heterogeneous media by analytic continuation. *Comm. Math. Phys.*, 90:473–491, 1983.

Although this widely cited paper does not mention sea ice, its relevance here arises from the amenability of sea ice to mathematical treatment as a heterogeneous composite material. It brings a rigorous mathematical formulation to the method of ‘analytic continuation’ for bounding effective parameters of composite materials. Useful background for this article may be found in [5], [6], [26].

[27] C. Orum, E. Cherkaev, and K. M. Golden. Recovery of inclusion separations in strongly heterogeneous composites from effective property measurements. *Proc. Royal Soc. Series A*,

468(2139):784–809, 2012.

Here 'strongly heterogeneous' refers to the high contrast between ice and brine in conductivity and electromagnetic permittivity. Similar to [15] in its general characterization of the inverse problem, it considers the problem of estimating structural parameters of composite media from an observed effective property. Many of the forward bounds obtained by the method of analytic continuation, e.g. [5], [6], [13], may be expressed as Möbius transformations whose coefficients are polynomial functions of the structural parameters. This paper presents a general technique for inverting such bounds to obtain a system of algebraic equations whose solution set bounds the region of admissible parameter values. The paper focuses on a separation parameter that quantifies the separation of brine inclusions in sea ice, building on the work of Bruno [7]. Data from [4] used in this paper is accessible as electronic supplementary material from the web pages of the Proceedings of the Royal Society.

## 8. Sea Ice Conductivity

[38] J. Zhu, K. M. Golden, A. Gully, and C. Sampson. A network model for electrical transport in sea ice. *Physica B (special issue for Proceedings of ETOPIM8)*, 405(14–15):3033–3036, 2010.

The authors develop a network model to describe the observed change in sea ice conductivity as a function of brine volume fraction.

[31] C. Sampson, K. M. Golden, A. Gully, and A. P. Worby. Surface impedance tomography for Antarctic sea ice. *Deep-Sea Research II*, 58:1149–1157, 2011.

This article examines conductivity data taken in Antarctica on first year sea ice using a classic four probe Wenner array. Vertical conductivity profiles are then reconstructed using $n$-layer inversion schemes.

[17] K. M. Golden, H. Eicken, A. Gully, M. Ingham, K. A. Jones, J. Lin, J. Reid, C. Sampson, and A. P. Worby. Critical behavior of electrical transport in sea ice. *Preprint*, 2012.

Two sets of sea ice conductivity data are evaluated. A strong electrical signature of brine connectivity in the microstructure of sea ice is detected at the 5% brine volume threshold. Percolation theory is used to accurately describe this phenomenon. The authors provide a rigorous basis for detecting the 'on-off switch' for fluid transport in sea ice using electrical methods.

## 9. Ice Floes

[19] A. Herman. Sea-ice floe-size distribution in the context of spontaneous scaling emergence in stochastic systems. *Phys. Rev. E*, 81(6):066123, 2010.

This paper points out the importance of characterizing the distribution of the sizes of the ice floes in the polar oceans. It is proposed that the scaled ice-floe diameters are well characterized by a special type of inverse-gamma distribution. This distribution is an emergent property of certain stochastic systems described by the generalized Lotka-Volterra (GLV) equation, and the author considers the possibility of developing a simple agent based GLV-type model for sea ice.

[20] A. Herman. Molecular-dynamics simulation of clustering processes in sea-ice floes. *Phys. Rev. E*, 84(5):056104, 2011.

The intermediate ice floe regime between freely drifting ice floes and compact ice cover is known as the medium concentration ice zone (MCIZ), which consists of strongly fragmented ice with clearly separated floes. The author develops a theoretical and numerical model of ice dynamics in the MCIZ, drawing on an analogy between sea ice in the MCIZ and two dimensional granular gasses. The model is applied to the observed clustering phenomenon of floes in the MCIZ.

## 10. Fluid Flow and the Navier-Stokes Equations

[8] Charles R. Doering and J. D. Gibbon. *Applied analysis of the Navier-Stokes equations.* Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge, 1995.

The first four chapters of this book serve as an introduction to the mathematics and physics of fluid flow and the Navier-Stokes equations. The remaining chapters give a nontechnical presentation of some of the mathematical issues related to existence and uniqueness of solutions of the Navier-Stokes equations.

## 11. Miscellaneous

[32] A. Stogryn and G. J. Desargant. The dielectric properties of brine in sea ice at microwave frequencies. *IEEE Trans. Antennas Propagat.*, AP-33(5):523–532, 1985.

The authors present a clever experiment that allows them to obtain and isolate the brine component of sea ice. After a careful analysis, the dielectric properties (or complex permittivities) of brine are computed at the microwave frequencies. Both a formula and graph are presented, which nicely depict the results.

[10] G. Frankenstein and R. Garner. Equations for determing the brine volume of sea ice from $-0.5^o$ to $-22.9^o$ C. *J. Glaciol.*, 6(48):943–944, 1967.

This article examines the relationship between the temperature, salinity, and brine volume fraction of sea ice. It gives a formula for calculating the brine volume fraction of a segment of sea ice from *in situ* temperature and salinity measurements.

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