Math 5490
October 1, 2014

Topics in Applied Mathematics:
Introduction to the Mathematics of Climate
Mondays and Wednesdays 2:30 – 3:45
http://www.math.umn.edu/~mcgehee/teaching/Math5490-2014-2Fall/
Streaming video is available at
http://www.ima.umn.edu/videos/
Click on the link: “Live Streaming from 305 Lind Hall”.
Participation:
https://umconnect.umn.edu/mathclimate

Glacial Cycles
Who was Milankovitch?
Milutin Milankovitch was a Serbian mathematician and professor at the University of Belgrade.
In 1920 he published his seminal work on the relation between insolation and the Earth’s orbital parameters.
In 1941 he published a book explaining his entire theory.
His work was not fully accepted until 1976.

Glacial Cycles
What happened in 1976?
“It is concluded that changes in the earth’s orbital geometry are the fundamental cause of the succession of Quaternary ice ages.”

Glacial Cycles
Solar Forcing (Hays, et al)

Glacial Cycles
Climate Response, Hays, et al
Three different temperature proxies from sea sediment data.

Glacial Cycles
Spectral Analysis of the Milankovitch cycles.
Glacial Cycles

Summer Solstice 65° N

Glacial Cycles

Milankovitch vs. Climate

Hays, et al, Summary

Forcing

Response

Increasing contribution

precession

eccentricity

obliquity

precession

Hays' explanation is that there are nonlinear feedbacks.

Are there other explanations?


Glacial Cycles

Climate Response (Zachos, et al)

Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41 Kyr and 100 Kyr.


Glacial Cycles

Hays, et al, Summary

Forcing

Response

Increasing contribution

precession

obliquity

eccentricity

precession

Nonlinear effects?

Other explanations?


Glacial Cycles

Spectral Analysis of the Climate Data

Nonlinear effects?

Other explanations?
**Spectral Analysis of the Climate Data**

Conclusion Remains: The Milankovitch cycles “pace” the Earth’s climate. Exactly how is not so clear.

**Zachos Summary (Revised)**

If we assume that glaciation depends on annual average insolation instead of insolation at summer solstice, then forcing and response are aligned.

**Something’s Missing**

Obliquity?

Data

Power spectrum

Glacial Cycles

Last Million Years is Different

A transition occurred about one million years ago: the amplitude increased and the dominant period changed from 41 kyr to 100 kyr.

-5 to 0 Myr

-5 to -1 Myr

-1 to 0 Myr

**What’s up with the Last Million Years?**

100,000 Year Problem: Why did the eccentricity signal become so dominant during the last million years?

400,000 Year Problem: If the last million years is dominated by eccentricity, what happened to the 400,000 year cycle?
(1, \( A \), \( BT \), \( C(T - \tau) \))

reduces to

\[ \frac{d\eta}{dt} = \varepsilon(T(q) - T) = h(q) \]

The function \( h \), and hence the equilibrium solution \( \eta^* \), depends on all the parameters of the Budyko model.

\[ h(e, \beta) = \frac{Q(e)}{\sqrt{1 - e^2}} \left[ 1 - \sqrt{1 - e^2 \cos^2 \beta - \frac{1}{2} \varepsilon \tan \beta \sin \beta} \right] \]

Therefore, the stable equilibrium ice line is a function of time:

\[ \eta^*(t) = \eta^*(e(t), \beta(t)) \]

Budyko’s model of ice-albedo feedback produces a climate response driven primarily by obliquity cycles, consistent with the dominance of obliquity in the climate data. The model fails to produce:

1. the amplitude changes over the past 5 million years, and
2. the frequency change 1 million years ago (‘mid-Pleistocene transition).

Hogg’s Model

\[ \frac{dT}{dt} = S(t) + G(C) - \sigma T \]

\[ \frac{dC}{dt} = W(C) + \beta (C_{\text{max}} - C) \max \left( \frac{dT}{dt} - \epsilon, 0 \right) \]

\[ S(t) = \sum_i \sin \left( \frac{2\pi t}{100} \right) \]

\[ G(C) = \theta + A \ln \left( \frac{C}{C_0} \right) \]

Glacial Cycles

Hogg’s Model

Hogg’s model shows how the carbon cycle can act as a feedback amplifying and modifying the insolation forcing, but the forcing is somewhat artificial, and the triggering mechanism is difficult to justify.

What if the 100,000 year glacial cycle is not driven by eccentricity, but is a natural oscillation of the Earth’s climate?

Saltzman and Maasch suggested just such a model.


Salzman-Maasch Model

\[ \dot{X} = -X - Y - a M(t) \]

\[ \dot{Y} = -p X + X Y + z Y - Z Y \]

\[ \dot{Z} = -q (X + Z) \]
The Salzman-Maasch model shows how the carbon cycle and the ocean currents can interact to produce unforced oscillations with periods of about 100,000 years. The same model with slightly different parameters can exhibit stationary behavior. By forcing the model with Milankovitch cycles and by slowly varying the parameters over the last two million years, they can produce a bifurcation from small oscillations tracking the Milankovitch cycles to large oscillations with a dominant 100,000 year period.

*Seems like a nice idea, but it is not widely accepted as the explanation, and it has some problems.*

The Hopf bifurcation explanation seems to have two serious problems ("cosmic coincidences").

1. Why does the intrinsic period of the glacial cycles just happen to have the same period as the eccentricity cycles?
2. Why does the phase of the glacial cycles agree with the phase of the obliquity and eccentricity cycles?