Low-order Climate Models

This is a list of 'simple' climate models. The papers are meant to be comprehensible & interesting to a general audience as well as relevant to climate researchers.

Energy-balance Models

1) Budyko, M.I. The effect of solar radiation variations on the climate of the Earth. *Tellus* **21**, 611-619 (1969).

Budyko's main concern is how variations in Earth's radiative forcing (caused by volcanic dust) affect the presence & growth of ice sheets on Earth. His model predicts that there exists a forcing threshold below which Earth jumps into a completely glaciated state. This state is now known as "Snowball Earth". In addition, a warm state might also be exist in which the Earth would be completely free of ice. Both scenarios have in fact occurred in Earth's past and it is remarkable that their basic physics is contained in such a low-order model. In contrast to other energy-balance models, Budyko assumes an atmosphere-ocean energy transport that is proportional to the difference of temperature with respect to global mean temperature. See Sellers or North (below) for different formulations which nevertheless do not destroy the Snowball bifurcation.

2) Sellers, W.D. A Global Climatic Model Based on the Energy Balance of the Earth-Atmosphere System. *Journal of Applied Meteorology* **8**, 392-400 (1969).

Sellers tries to account in a lot more detail for the meridional energy transport in atmosphere and ocean than Budyko. Nevertheless his model contains the same energy-balance equation as Budyko's and basically also linearizes outgoing longwave radiation. Like Budyko, Sellers finds a bifurcation that leads to a Snowball state. It is worth noting that Sellers considers both fixed and temperature-dependent albedos; with fixed albedo the Snowball state disappears. This underlines how important the ice-albedo behavior is for the Snowball; for a recent paper that uses a slightly more detailed albedo function see Abbot et al "The Jormungand Global Climate State and Implications for Neoproterozoic Glaciations" (2011). Note that neither Sellers nor Budyko account for albedo changes due to clouds; for a discussion of how clouds might affect Snowball initiation/deglaciation, see Pierrehumbert (below).

3) North, G.R. Analytical Solution to a Simple Climate Model with Diffusive Heat Transport. *Journal of the Atmospheric Sciences* **32**, 1301-1307 (1975).

This paper is notable for its clarity and concision. It contains a discussion of a Budyko-like energy-balance model and presents an analytical solution. North mostly adapts Budyko's model but assumes that meridional heat transport is diffusive. An important result is that the Snowball and 'warm' equilibrium solutions of the model coexist over a big range of radiative forcing. North also analyzes the stability of the equilibrium solutions and shows that intermediate glaciations are unstable.

4) North, G.R. Theory of Energy-Balance Climate Models. *Journal of the Atmospheric Sciences* **32**, 2033-2043 (1975).

This paper is most interesting to read for its use of an expansion in Legendre polynomials that can also be applied to other planetary-scale problems (also see Stone below). North expands the solution of an energy-balance model in even Legendre polynomials and argues that the first two terms capture most of the model's behavior. North also shows that the two-term solution with a globally constant diffusivity amounts to the same heat transport as that assumed by Budkyo.

Atmospheric Column Models

6) Manabe, S. & Strickler, R.F. Thermal Equilibrium of the Atmosphere with a Convective Adjustment. *Journal of the Atmospheric Sciences* **21**, 361-385 (1964).

7) Manabe, S. & Wetherald, R.T. Thermal Equilibrium of the Atmosphere with a Given Distribution of Relative Humidity. *Journal of the Atmospheric Sciences* **24**, 241-259 (1967).

These papers present some of the first calculations done with a radiative column model. Even though this kind of model is far less 'simple' than Budyko-Sellers type models, they are still much simpler than full global climate models and widely used in research (an accessible modern column model that uses a Python interface can be found here: http://people.su.se/~rcaba/climt/). For a historical overview of early radiative calculations and models, see: http://www.aip.org/history/climate/Radmath.htm.

Manabe & Strickler (1964) describe their model in detail and illustrate for example the treatment of convective adjustment and the radiative effect of clouds. Modern column models still have a comparable structure, which makes this paper worth reading. Manabe & Weatherald (1967) apply this model and present one of the first complex calculations which demonstrated the greenhouse effect of CO_2.

8) Kasting, J.F., Whitmire, D.P. & Reynolds, R.T. Habitable Zones around Main Sequence Stars. *Icarus* **101**, 108-128 (1993).

Kasting & Whitmire use a radiative column model to compute the inner & outer limits of the habitable zone. This illustrates how such a model can be applied to explore both the runaway greenhouse scenario as well as the outer edge where a planet irreversibly freezes over due to CO_2 condensation.

'Non-classical' Simple Models

9) Pierrehumbert, R.T. The hydrologic cycle in deep-time climate problems. *Nature* **419**, 191-198 (2002).

This paper presents a simple scaling argument for how latent heat transport might have been different in much colder or much warmer climates. Pierrehumbert also presents calculations with a radiative column model to explore how clouds affect a Snowball state. The last section discusses boundary-layer dynamics and how this affects precipitation and therefore the CO_2 silicate-weathering feedback.

10) Stone, P.H. Constraints on dynamical transports of energy on a spherical planet. *Dynamics of Atmospheres and Oceans* **2**, 123-139 (1978).

Stone uses an expansion in Legendre Polynomials to constrain what shape the total energy transport of the atmosphere & ocean should have as well as its magnitude. For a modern discussion of this 'model', see Section 3 in Enderton & Marshall “Explorations of Atmosphere–Ocean–Ice Climates on an Aquaplanet and Their Meridional Energy Transports” (2009).

11) Emanuel, K. A simple model of multiple climate regimes. *J. Geophys. Res.* **107**, 10 PP. (2002).

Emanuel constructs a two-box model that includes several feedback mechanisms different from the ice-albedo feedback of Budyko-Sellers type models. Notably, this model produces multiple stable states and hysteresis between warm and cold climates.