Math 5490

October 20, 2014

Topics in Applied Mathematics: Introduction to the Mathematics of Climate

Mondays and Wednesdays 2:30 – 3:45

http://www.math.umn.edu/~mcgehee/teaching/Math5490-2014-2Fall/

Streaming video is available at
http://www.ima.umn.edu/videos/
Click on the link: "Live Streaming from 305 Lind Hall".

Participation:
https://umconnect.umn.edu/mathclimate

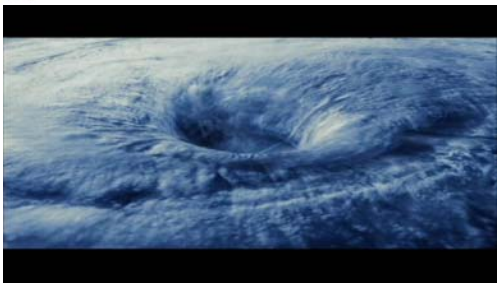
MCRN Mathematics and Climate Research Network

IMA Institute for Mathematics and Its Applications

Good Science or Bad Science?

Bad Science

Thermodynamics Violated



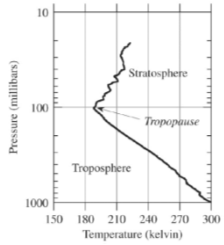
The Day After Tomorrow

Bad Science

Thermodynamics Violated

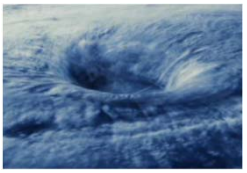
"It's drawing −150º air down from the upper troposphere."

March 15 1993, 12Z
2.00° S 169.02° W



Pierrehumbert, *Principles of Planetary Climate*

The Day After Tomorrow



Upper troposphere: ~ 190 K

The Day After Tomorrow:

-150 °F = −101 °C = 172 K

Only a slight exaggeration.

Bad Science

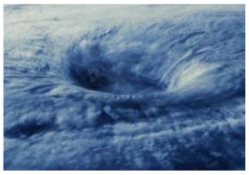
Thermodynamics Violated

Professor Hall:
"It's drawing air −150º air down from the upper troposphere."

Professor Rapson:
"Wouldn't it heat up before it reached the surface?"

Professor Hall:
"No, it's descending too fast."

Kate Meyer:
"Wouldn't that violate entropy?"

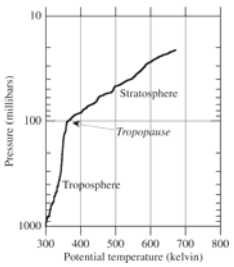


The Day After Tomorrow

Bad Science

Thermodynamics Violated

March 15 1993, 12Z
2.00° S 169.02° W



Pierrehumbert, *Principles of Planetary Climate*

Bringing the air down from the upper troposphere involves increasing the pressure from 0.1 atmosphere to 1 atmosphere, thereby heating it.

Potential temperature: The temperature the air would be if compressed to 1 atmosphere.

Potential temperature of the upper troposphere:

350 K = 77 °C = 171 °F

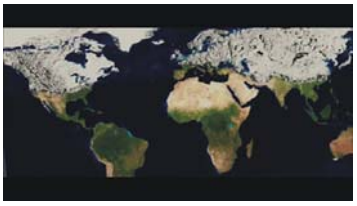
Definitely would not freeze the fuel lines of RAF helicopters.

Bad Science

Energy Balance Violated

Most of the northern hemisphere land covered with 30 feet of snow, converted to about 1 meter of water.

Let's say half of all the land, or 15% of the Earth's surface.



70% of the surface is ocean, so about 15/70 or 0.2 meters of ocean evaporated and turned to snow in a few days.

Where did the energy go?

Bad Science

Energy Balance Violated

Table S1. Planetary Heat Storage: Ocean, Ice, Air and Land.

Energy required to melt ice and warm the air, land and ocean by specified amounts.[1]

Ocean warming by 1°C through 1 km depth of ocean. Heat storage is $1°C \times 10^3$ g/cm^2 $\times$ 1 cal/g $\times$ 4.19 joules/cal $\times$ area Earth $\times$ 0.7 ~ 15×10^{23} joules ~ **93 W yr/m^2**.

Ice sheet melting to raise sea level 1 meter. Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C). Energy required is 100 cal/g (80 cal/g for melting). Energy for 1 meter of sea level: 100g/cm^2 × 100cal/g × 4.19 joules/cal × area Earth × 0.7 ~ 1.5×10^{23} joules ~ **9.3 W yr/m^2**.

Sea ice melting (all sea ice on planet). Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C), and that sea ice covers 4% of the planet with mean thickness 2.5 m. Energy required is 250 g cm^2 × 100 cal/g (80 cal/g for melting) × 4.19 joules/cal × 0.04 × area Earth ~ 2.14×10^{22} joules ~ **1.3 W yr/m^2**.

Air warming by 1°C. The Earth's atmospheric mass is ~ 10 m of water. Heat capacity of air ~ 0.24 cal/g°C. Energy to raise air temperature 1°C: 1°C × 1000 g/cm^2 × 0.24 cal/g°C × 4.19 joules/cal × area Earth ~ 0.26×10^{22} joules ~ **0.32 W yr/m^2**.

Land surface warming by 1°C. The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~10 m. With density ~ 3 g/cm^3, heat capacity ~ 0.2 cal/g°C, and 0.29 fractional land coverage, land heat storage is 10^7 cm ×3 g/cm^3 × 0.2 cal/g°C × 1°C ×4.19 joules/cal × area Earth × 0.29 ~ 0.37×10^{22} joules ~ **0.23 W yr**. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ **0.7 W yr/m^2**.]

[1]Note that 1 W sec = 1 joule, # sec/year ~ $\pi \times 10^7$, area Earth ~ 5.1×10^{18} cm^2, 1 W yr over full Earth ~ 1.61×10^{22} joules, ocean fraction of Earth ~ 0.7, 1 calorie ~ 4.19 joules.

James Hansen, et al, *Earth's Energy Imbalance: Confirmation and Implications*, SCIENCE 308 (2005), p. 1431

Bad Science

Energy Balance Violated

About 0.2 meters of the ocean evaporated and turned to snow in a few days.

It takes 9.3 Wyr/m^2 to turn glaciers into 1 meter of ocean, so turning 0.1 meter of ocean into snow (at –10°C) would dissipate 0.93 Wyr/m^2. To accomplish that in 3 days would have to dissipate more than 100 W/m^2.

Recall also that the North Atlantic Ocean temperature dropped by "13 degrees" during the same time. Let's say that, averaged over the entire ocean, that amounted to one degree (Celsius) for the top 100 meters.

Ocean warming by 1°C through 1 km depth of ocean. Heat storage is $1°C \times 10^3$ g/cm^2 $\times$ 1 cal/g $\times$ 4.19 joules/cal $\times$ area Earth $\times$ 0.7 ~ 15×10^{23} joules ~ **93 W yr/m^2**.

James Hansen, et al, SCIENCE 308 (2005), p. 1431

Cooling the ocean would take 9.3 Wyr/m^2, which over 3 days would be a heat imbalance of -1000 W/m^2.

Recall that the current outgoing long wave radiation for the Earth is 235 W/m^2.

Heat Imbalance

Table S1. Planetary Heat Storage: Ocean, Ice, Air and Land.

Energy required to melt ice and warm the air, land and ocean by specified amounts.[1]

Ocean warming by 1°C through 1 km depth of ocean. Heat storage is $1°C \times 10^3$ g/cm^2 $\times$ 1 cal/g $\times$ 4.19 joules/cal $\times$ area Earth $\times$ 0.7 ~ 15×10^{23} joules ~ **93 W yr/m^2**.

Ice sheet melting to raise sea level 1 meter. Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C). Energy required is 100 cal/g (80 cal/g for melting). Energy for 1 meter of sea level: 100g/cm^2 × 100cal/g × 4.19 joules/cal × area Earth × 0.7 ~ 1.5×10^{23} joules ~ **9.3 W yr/m^2**.

Sea ice melting (all sea ice on planet). Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C), and that sea ice covers 4% of the planet with mean thickness 2.5 m. Energy required is 250 g cm^2 × 100 cal/g (80 cal/g for melting) × 4.19 joules/cal × 0.04 × area Earth ~ 2.14×10^{22} joules ~ **1.3 W yr/m^2**.

Air warming by 1°C. The Earth's atmospheric mass is ~ 10 m of water. Heat capacity of air ~ 0.24 cal/g°C. Energy to raise air temperature 1°C: 1°C × 1000 g/cm^2 × 0.24 cal/g°C × 4.19 joules/cal × area Earth ~ 0.26×10^{22} joules ~ **0.32 W yr/m^2**.

Land surface warming by 1°C. The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~10 m. With density ~ 3 g/cm^3, heat capacity ~ 0.2 cal/g°C, and 0.29 fractional land coverage, land heat storage is 10^7 cm ×3 g/cm^3 × 0.2 cal/g°C × 1°C ×4.19 joules/cal × area Earth × 0.29 ~ 0.37×10^{22} joules ~ **0.23 W yr**. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ **0.7 W yr/m^2**.]

[1]Note that 1 W sec = 1 joule, # sec/year ~ $\pi \times 10^7$, area Earth ~ 5.1×10^{18} cm^2, 1 W yr over full Earth ~ 1.61×10^{22} joules, ocean fraction of Earth ~ 0.7, 1 calorie ~ 4.19 joules.

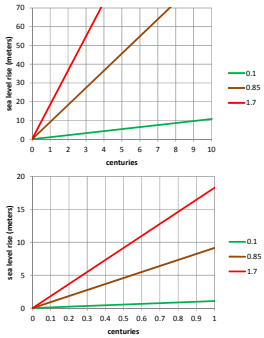
James Hansen, et al, *Earth's Energy Imbalance: Confirmation and Implications*, SCIENCE 308 (2005), p. 1431

Heat Imbalance

Suppose that all the heat imbalance went to melting the glaciers.

It takes 9.3 Wyr/m^2 to turn glaciers into 1 meter of ocean. If the heat imbalance is w W/m^2, the sea level would rise at the rate of w/9.3 meters per year. At the current imbalance of 0.85 W/m^2, the rate is about 0.091 meters per year, or 9.1 meters per century.

Melting all the glaciers would cause a sea level rise of about 70 meters and would take about 760 years at the current imbalance.

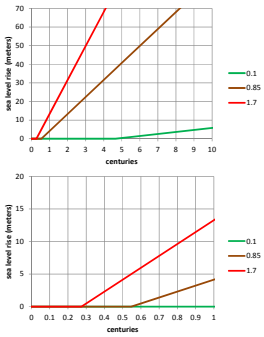


Heat Imbalance

Suppose now that all the heat imbalance first goes to raising the top kilometer of ocean by 0.5 °C, and then goes to melting the glaciers.

It takes 46.5 Wyr/m^2 to raise the temperature of a kilometer of ocean by 0.5 °C. If the heat imbalance is w W/m^2, the increase would be achieved in 46.5/w years, after which the sea level would rise at w/9.3 meters per year.

At the current imbalance of 0.85 W/m^2, the ocean temperature increase would delay the sea level rise by about 56 years.

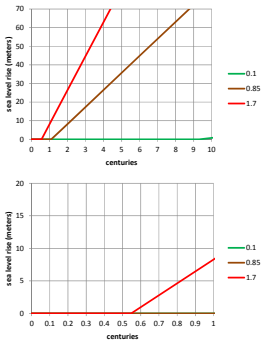


Heat Imbalance

Suppose instead that all the heat imbalance first goes to raising the top kilometer of ocean by 1 °C, and then goes to melting the glaciers.

It takes 93 Wyr/m^2 to raise the temperature of a kilometer of ocean by 1 °C. If the heat imbalance is w W/m^2, the increase would be achieved in $93/w$ years, after which the sea level would rise at $w/9.3$ meters per year.

At the current imbalance of 0.85 W/m^2, the ocean temperature increase would delay the sea level rise by about 112 years.



Heat Imbalance

Questions for the Coming Centuries

How will the heat imbalance be divided between heating the ocean and melting the glaciers?

How will the heat imbalance be affected by increasing atmospheric greenhouse gases?

How will the heat imbalance be affected by increasing ocean temperatures?

What happens to the weather as the ocean temperature rises and the ice caps melt?

What should we do about coastal cities?

Good Science

Dansgaard-Oeschger Events

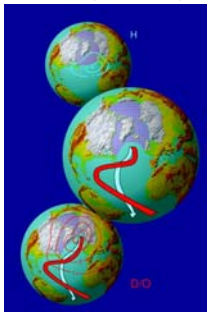
"Global warming" can cause the Northern Hemisphere to cool.

Melting glaciers can lower the salinity of the North Atlantic, causing a decrease in the flow of the Atlantic Meridional Overturning Circulation (AMOC), slowing the heat transfer to the Northern Hemisphere.

This phenomenon is believed to have caused the Younger Dryas.

Heat Imbalance

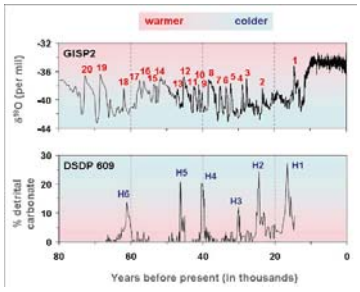
Heinrich and Dansgaard-Oeschger events



http://www.pik-potsdam.de/~stefan/sampleimages.html

Heat Imbalance

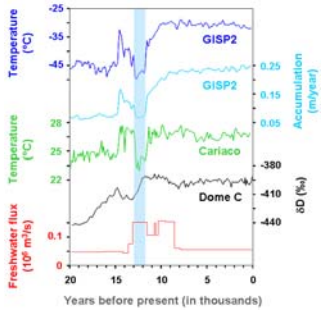
Heinrich and Dansgaard-Oeschger events



http://www.ncdc.noaa.gov/paleo/abrupt/data3.html

Heat Imbalance

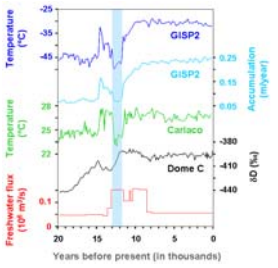
The Younger Dryas



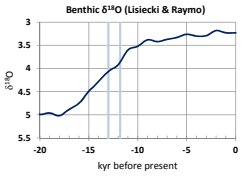
http://www.ncdc.noaa.gov/paleo/abrupt/data4.html

Heat Imbalance

The Younger Dryas



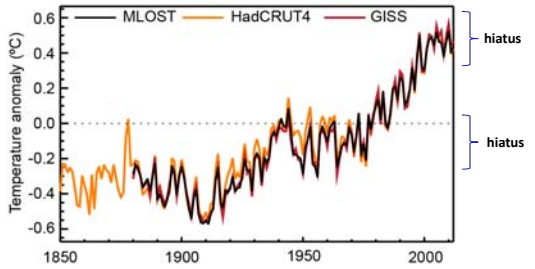
Only a minor impact on ice volume.



http://www.ncdc.noaa.gov/paleo/abrupt/data4.html

Heat Imbalance

What's Happening Now?

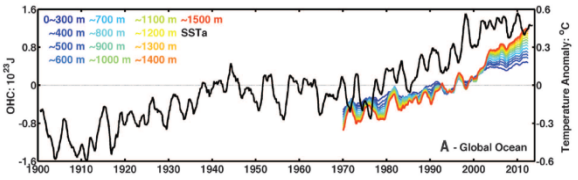


IPCC AR5 (2013) Figure 2.20

Heat Imbalance

What's Happening Now?

The heat imbalance is being absorbed by the ocean (at 1000 meters, not the surface).



Chou & Tung, Science 345 (2014) p 897

Heat Imbalance

What's Happening Now?

The Good News

The surface temperatures are remaining fairly constant, so the perceived warming is small.

The Bad News

The heat is still there and will take a long time to dissipate and may have other effects, *e.g.* hurricanes, biological changes.

If the surface temperature does not rise, the heat imbalance will remain high, leading eventually to more warming.

There is evidence that hiatuses (hiati?) correspond to 60 year cycles of the AMOC. Will we experience another strong warming period in 30 years?

Chou & Tung, Science 345 (2014) p 897