Math 5490
October 1, 2014

Topics in Applied Mathematics: Introduction to the Mathematics of Climate

Mondays and Wednesdays 2:30 – 3:45

http://www.math.umn.edu/~mcgehee/teaching/Math5490-2014-2Fall/

Streaming video is available at
http://www.ima.umn.edu/videos/
Click on the link: "Live Streaming from 305 Lind Hall".

Participation:
https://umconnect.umn.edu/mathclimate

MCRN Mathematics and Climate Research Network
IMA Institute for Mathematics and Its Applications

Glacial Cycles

Who was Milankovitch?

Milutin Milankovitch was a Serbian mathematician and professor at the University of Belgrade.

In 1920 he published his seminal work on the relation between insolation and the Earth's orbital parameters.

In 1941 he published a book explaining his entire theory.

His work was not fully accepted until 1976.


Milutin Milankovitch
1879-1958

Glacial Cycles

What happened in 1976?


James D. Hays


John Imbrie

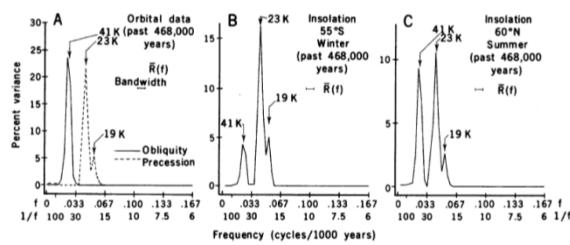
Hays, Imbrie, and Shackleton, "Variations in the Earth's Orbit: Pacemaker of the Ice Ages," *Science* **194**, 10 December 1976.

"It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages."


Nicholas Shackleton

Glacial Cycles

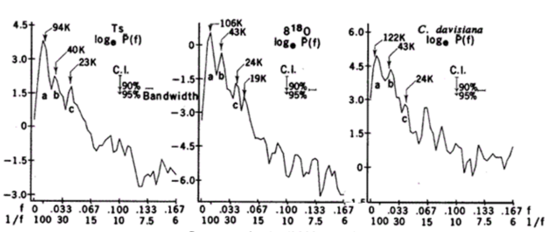
Solar Forcing (Hays, et al)



Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Climate Response, Hays, et al

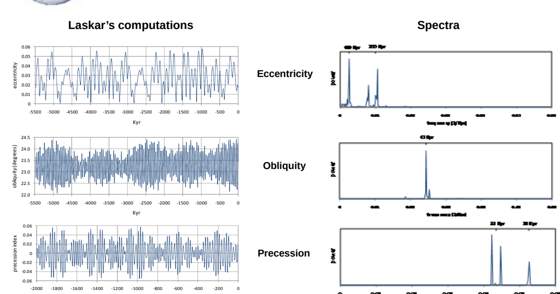


Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1125

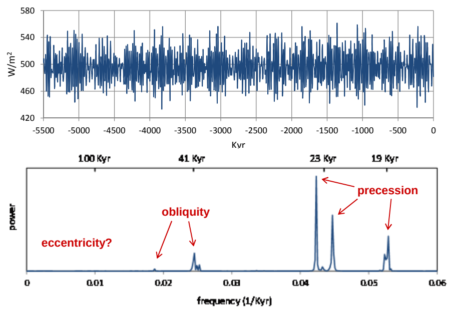
Glacial Cycles

Spectral Analysis of the Milankovitch cycles.



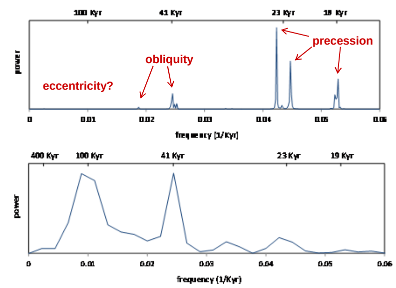
Glacial Cycles

Summer Solstice 65°N



Glacial Cycles

Milankovitch vs. Climate



Glacial Cycles

Hays, et al, Summary

	Forcing		Response	
Increasing contribution ↑	precession		eccentricity	↑
	obliquity		obliquity	
	eccentricity		precession	

Hays' explanation is that there are nonlinear feedbacks.

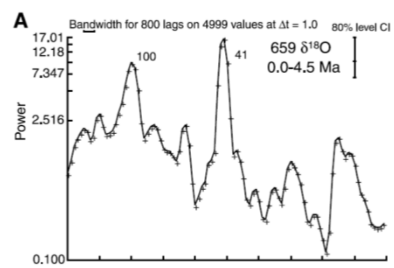
Are there other explanations?

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

Climate Response (Zachos, et al)

Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Zachos, et al, Summary

	Forcing		Response	
Increasing contribution ↑	precession		obliquity	↑
	obliquity		eccentricity	
	eccentricity		precession	

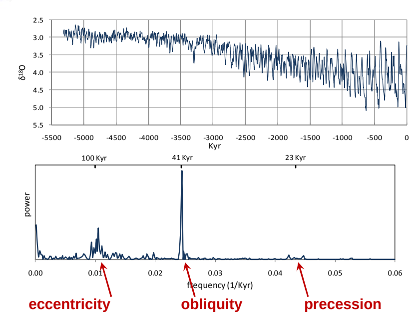
Nonlinear effects?

Other explanations?

Zachos, *et al*, *Science* **292** (2001), p. 689

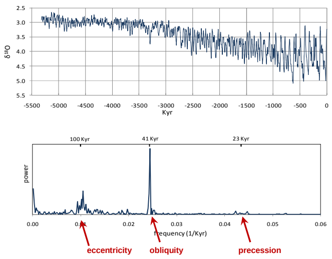
Glacial Cycles

Spectral Analysis of the Climate Data



Glacial Cycles

Spectral Analysis of the Climate Data



Conclusion Remains:
The Milankovitch cycles "pace" the Earth's climate.

Exactly how is not so clear.

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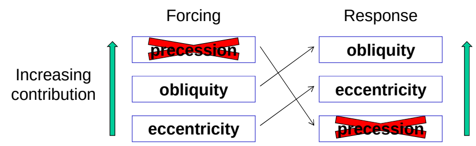
Glacial Cycles

Zachos Summary (Revised)

If we assume that glaciation depends on annual average insolation instead of insolation at summer solstice, then forcing and response are aligned.



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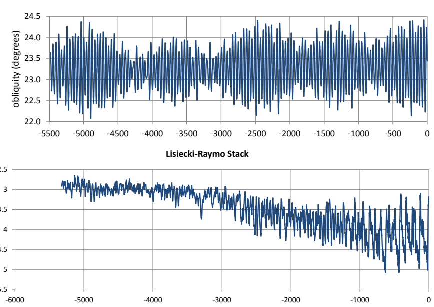
Glacial Cycles

Something's Missing

obliquity

climate



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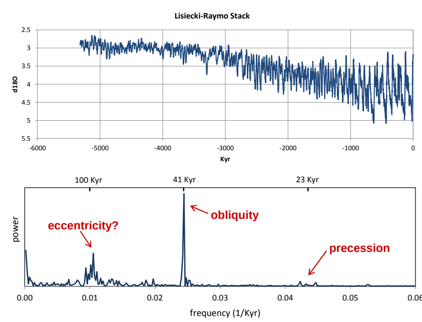
Glacial Cycles

Something's Missing

data

power spectrum



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Glacial Cycles

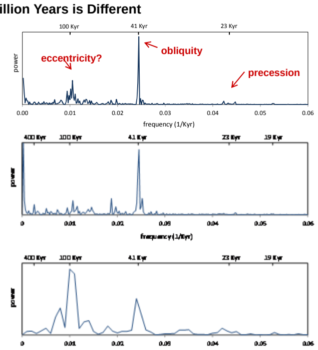
Last Million Years is Different

A transition occurred about one million years ago: the amplitude increased and the dominant period changed from 41 kyr to 100 kyr.

-5 to 0 Myr

-5 to -1 Myr

-1 to 0 Myr



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Glacial Cycles

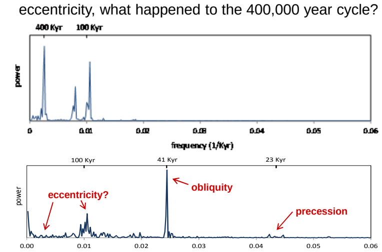
What's up with the Last Million Years?

100,000 Year Problem: Why did the eccentricity signal become so dominant during the last million years?

400,000 Year Problem: If the last million years is dominated by eccentricity, what happened to the 400,000 year cycle?

eccentricity

data



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Glacial Cycles

Budyko's Model

surface temperature, sin(latitude), ice line, $\bar{T}=\int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\bar{T}-T)$$

heat capacity, insolation, albedo, OLR, heat transport

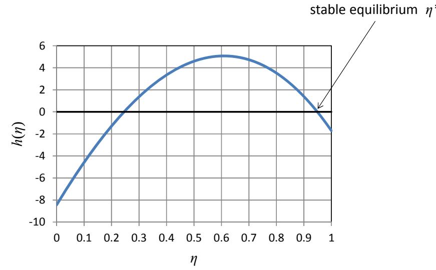
reduces to

$$\frac{d\eta}{dt}=\varepsilon\left(T(\eta)-T_c\right)\equiv h(\eta)$$

Glacial Cycles

Budyko's Model

$$\frac{d\eta}{dt}=h(\eta)$$



Glacial Cycles

Budyko's Model

$$\frac{d\eta}{dt}=h(\eta)$$

The function h, and hence the equilibrium solution η^*, depends on all the parameters of the Budyko model.

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\bar{T}-T)$$

In particular, η^* depends on Q and $s(y)$, which depend on the eccentricity e and the obliquity β.

$$Q(e)=\frac{Q_0}{\sqrt{1-e^2}}$$

$$s(y,\beta)=\frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\theta-y\cos\beta\right)^2}\,d\theta$$

$$\eta^*=\eta^*(e,\beta)$$

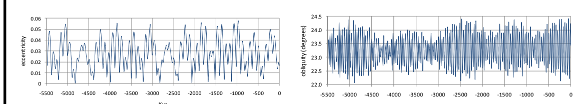
McGehee & Lehman, A Paleoclimate Model of Ice-Albedo Feedback Forced by Variations in Earth's Orbit, SIAM J. Applied Dynamical Systems **11** (2012), 684–707.

Glacial Cycles

Budyko's Model

$$\frac{d\eta}{dt}=h(\eta,e,\beta)$$

The eccentricity e and the obliquity β are given by Laskar as functions of time.



Therefore, the stable equilibrium ice line is a function of time:

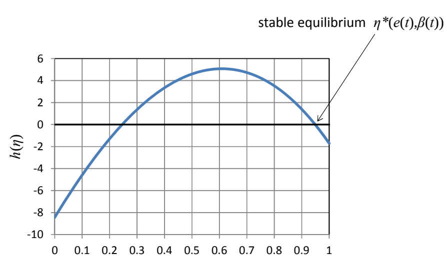
$$\eta^*(t)=\eta^*(e(t),\beta(t))$$

McGehee & Lehman, A Paleoclimate Model of Ice-Albedo Feedback Forced by Variations in Earth's Orbit, SIAM J. Applied Dynamical Systems **11** (2012), 684–707.

Glacial Cycles

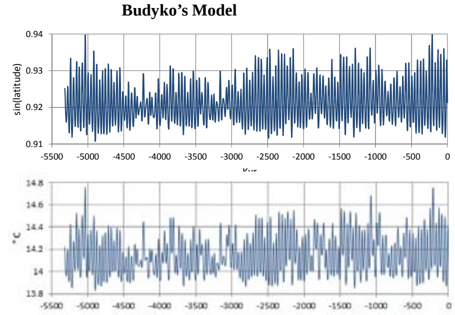
Budyko's Model

$$\frac{d\eta}{dt}=h(\eta,e,\beta)$$



Glacial Cycles

Budyko's Model



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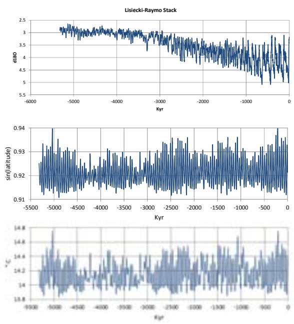
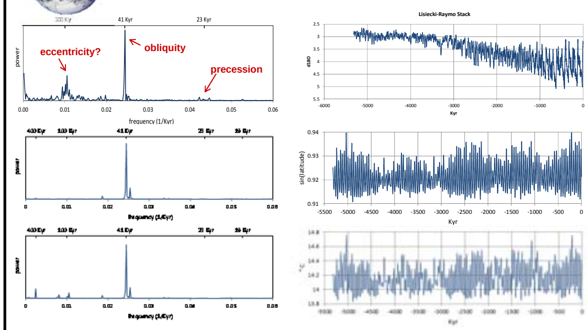
Glacial Cycles

Budyko's Model



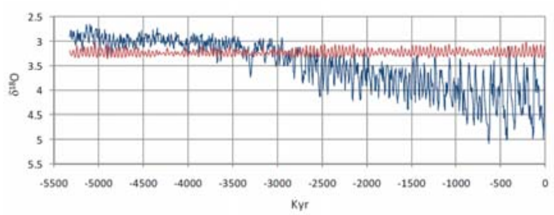
Glacial Cycles

Budyko's Model



Glacial Cycles

Budyko's Model



red: model simulation
blue: data

McGehee & Lehman, SIAM J. Applied Dynamical Systems **11** (2012), 684–707.

Glacial Cycles

Budyko's Model

Budyko's model of ice-albedo feedback produces a climate response driven primarily by obliquity cycles, consistent with the dominance of obliquity in the climate data.

The model fails to produce:

1. the amplitude changes over the past 5 million years, and
2. the frequency change 1 million years ago ("mid-Pleistocene transition).

Glacial Cycles

Budyko's Model

MATLAB Program:
PaleoBudyko

Download from
http://www.math.umn.edu/~mcgehee/Software/

Glacial Cycles

Some Recent Developments
Modeling Glacial Cycles

Glacial Cycles

Hogg's Model

surface temperature

$$c\frac{dT}{dt} = S(t) + G(C) - \sigma T^4,$$

atmospheric carbon

$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta(C_{\max} - C)\max\left(\frac{dT}{dt} - \varepsilon, 0\right).$$

weathering — CO2 outgassing — volcanos

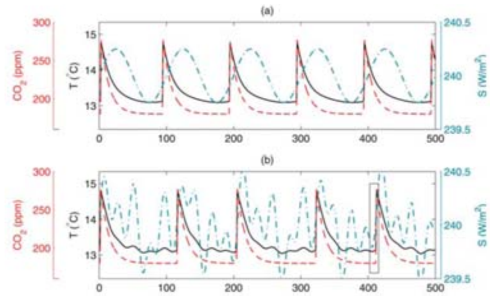
$$S(t) = \bar{S} + \sum_i S_i \sin\left(\frac{2\pi t}{\Gamma_i}\right) \quad \text{insolation}$$

$$G(C) = \bar{G} + A\ln\left(\frac{C}{C_0}\right) \quad \text{greenhouse forcing}$$

Andrew McC. Hogg, "Glacial cycles and carbon dioxide: A conceptual model," *Geophysical Research Letters* **35** (2008).

Glacial Cycles

Hogg's Model



Glacial Cycles

Hogg's Model

Hogg's model shows how the carbon cycle can act as a feedback amplifying and modifying the insolation forcing, but the forcing is somewhat artificial, and the triggering mechanism is difficult to justify.

What if the 100,000 year glacial cycle is not driven by eccentricity, but is a natural oscillation of the Earth's climate?

Saltzman and Maasch suggested just such a model.

Glacial Cycles

Salzman-Maasch Model

Milankovitch forcing

global ice mass $\longrightarrow$ $\dot{X} = -X - Y - uM(t)$

atmospheric CO_2 $\longrightarrow$ $\dot{Y} = -pZ + rY + sZ^2 - Z^2Y$

ocean circulation $\longrightarrow$ $\dot{Z} = -q(X + Z)$

Barry Salzman and Kirk A. Maasch, "A Low-Order Dynamical Model of Global Climatic Variability Over the Full Pleistocene," *Journal of Geophysical Research* **95** (D2), 1955-1963 (1990)

Glacial Cycles

Salzman-Maasch Model

unforced

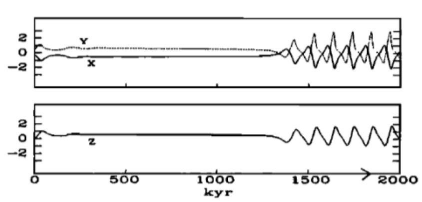


Fig. 7. Unforced (free) solution for (4)–(6) with $q = 1.2$, $s = 0.8$, and p and r varying linearly between $0.8 \rightarrow 1.0$ and $0.7 \rightarrow 0.8$, respectively. (Top) Model solutions for X (solid) and Y (dashed). (Bottom) Model solution for Z.

Glacial Cycles

Salzman-Maasch Model

forced

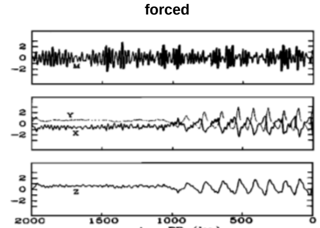


Fig. 8. Forced solution for (4)–(6) with $q = 1.2$, $s = 0.8$, $u = 0.7$, and p and r varying linearly between $0.8 \rightarrow 1.0$ and $0.7 \rightarrow 0.8$, respectively. (Top) Normalized 65°N July insolation curve (M) used as forcing. (Middle) Model solutions for X (solid) and Y (dashed). (Bottom) Model solution for Z.

Glacial Cycles

Salzman-Maasch Model

The Salzman-Maasch model shows how the carbon cycle and the ocean currents can interact to produce unforced oscillations with periods of about 100,000 years. The same model with slightly different parameters can exhibit stationary behavior. By forcing the model with Milankovitch cycles and by slowly varying the parameters over the last two million years, they can produce a bifurcation from small oscillations tracking the Milankovitch cycles to large oscillations with a dominant 100,000 year period.

Seems like a nice idea, but it is not widely accepted as the explanation, and it has some problems.

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Glacial Cycles

Salzman-Maasch Model

The Hopf bifurcation explanation seems to have two serious problems ("cosmic coincidences").

1. Why does the intrinsic period of the glacial cycles just happen to have the same period as the eccentricity cycles?
2. Why does the phase of the glacial cycles agree with the phase of the obliquity and eccentricity cycles?

Samantha Oestreicher, PhD Thesis, 2014.

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