Phase Transitions in Permafrost and Abrupt Arctic Climate Change

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Introduction

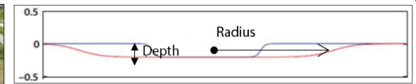
➢ The permafrost methane emission problem is the focus of attention in different climate models. We present a mathematical model for permafrost lake methane emission and influence its on the climate system.

➢ We model this process using the theory of nonlinear phase transitions for permafrost. Further, we find that a climate catastrophe possibility depends on a value of feedback connecting the methane concentration in the atmosphere and temperature, and on the tundra permafrost methane pool.

➢ We note that the permafrost lake model that we developed for the methane emission positive feedback loop problem is a conceptual climate model.

Permafrost Lake Growth Model

As a result of tundra permafrost thawing, a number of small lakes have formed and extended, and methane has entered the atmosphere



➢ Typically, the lakes have horizontal sizes of 100-2000 m, and a small depth.

➢ We can assume locally that the lake boundary is a sphere with a large radius of curvature *R*.

➢ Therefore, the lakes are similar to a shallow large bowl. The growth of the lakes is a slow process.

➢ Mathematically, permafrost thawing can be described by the classical Stefan approach.

Stefan-type models as limiting cases of the phase field equations (Caginalp, 1989)

Phase field equations

$$u_t = K\Delta u - \frac{b}{2}\varphi_t$$

$$\alpha\xi^2\varphi_t = \xi^2\Delta\varphi + a^{-1}g(\varphi) + 2(u-\theta)$$

Scaling limit:

$\alpha, \xi \to 0 \quad \alpha = \text{fixed}$

$\xi a^{-1/2} \to 0$

⟷ **Classical Stefan model**

$$u_t = K\Delta u$$

$$\left.\begin{aligned} bv &= K(\nabla u_S - \nabla u_L)\cdot \mathbf{n} \\ u &= 0 \end{aligned}\right\} \text{ on } \Gamma$$

Scaling limit:

$\alpha, \xi \to 0 \quad \alpha = \text{fixed}$

$\xi a^{-1/2} = \text{fixed}$

⟷ **Modified Stefan model**

$$u_t = K\Delta u$$

$$\left.\begin{aligned} bv &= K(\nabla u_S - \nabla u_L)\cdot \mathbf{n} \\ \Delta S(u-\theta) &= -\sigma\kappa - \alpha\sigma v \end{aligned}\right\} \text{ on } \Gamma$$

u – phase temperature (indexes S and L – solid and liquid states, respectively); g – Ginzburg-Landau potential; φ – order parameter (-1 or 1); θ – phase transition temperature; K – thermal diffusivity; b – dimensionless latent heat; α, ξ, a – dimensionless parameters which can be find experimentally; $\mathbf{n}$ – the unit normal vector with respect to the thawing front; v – the normal front velocity; κ – the front curvature; ΔS – a parameter describing a difference between thermodynamics of ice and water; Γ – separating surface; σ – surface tension.

Phenomenological model for lake growth

the front velocity $\quad v(x,y,z,t) = -\mu\kappa(x,y,z,t)$

the lake radius $\quad \frac{dR}{dt} = \delta - \mu R^{-1}$

v – the normal thawing front velocity at the point (x; y; z); κ – the mean front curvature at this point; μ – positive coefficient; t – time; R – lake radius; δ – function of the microscopic parameters of the model.

Stochastic effects in model for lake growth

the Fokker-Planck equation $\quad \frac{\partial(f(R)\rho)}{\partial R} = d\frac{\partial^2\rho}{\partial R^2}$

the Pareto law $\quad \rho(A) = kA_{\min}^m A^{-(m+1)}, \quad A \in [A_{\min}, \infty)$

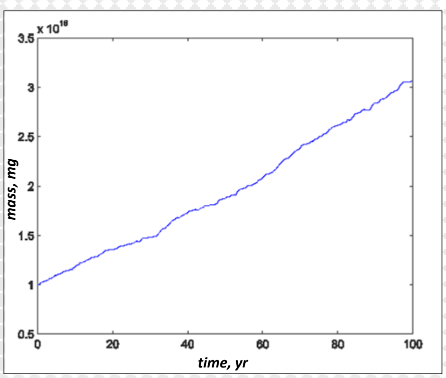
ρ – function for time depending lake radius; d – coefficient of the diffusion process of lake distribution; k – positive coefficient; A_{min} – a minimal lake size; m – parameter related to μ.

Methane emission

the permafrost lake methane emission rate $\quad V = \beta(BR_{av} - 1)\exp(-b_0 / u_{av}(t))$

empirical formula for methane fluxes (Frolking & Grill, 1994) $\quad F = \exp(0.492 + 0.126(T - 273) - 0.057W)$

V – the total rate of the methane emission; R_{av} – the averaged lake radius; u_{av} – the average temperature; β, b_0, B – positive constants; F – measured fluxes; T – temperature on the depth; W – water level in the lake.



Numerical experiment 1: A linear growth of methane mass (y-axis) in case of a small feedback coefficient (x-axis – time)

The positive feedback in 'toy' climate model

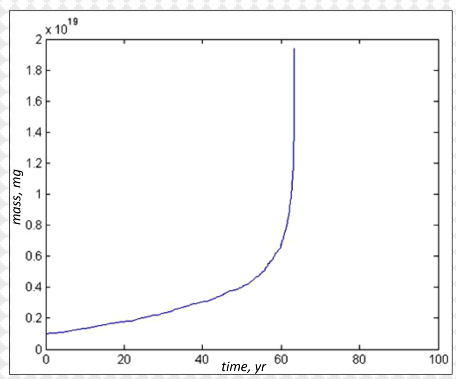
$$u(t) = u_{av} + \gamma X$$

u – climate system temperature; u_{av} – a mean temperature for the summer season without warming effect; X – the total methane mass in atmosphere; γ – the feedback coefficient.

Methane mass changing rate

$$\frac{dX}{dt} = H(t,X) + \eta X$$

η – the averaged observed methane growth; H – methane emission just for the summer season.



Numerical experiment 2: A sharp increasing of methane mass (y-axis) in case of a large feedback coefficient (x-axis – time)

Conclusions

➢ We propose a simple method to compute methane emission from permafrost, using a natural assumption that the horizontal dimensions of the lakes are much larger than the lake depth.

➢ The numerical simulations based on these theoretical results show that there are two different possible scenarios for methane emission. The first case appears if the feedback coefficient, that connects the temperature at lake surfaces and methane concentration is small. Then the methane concentration growth over time can be described by a linear function.

➢ The most interesting situation occurs if the feedback coefficient is large. If this occurs, we observe linear growth on an initial time interval, then an increasingly parabolic curve, which finally this transforms into a sharply increasing curve, that can be interpreted as a "Arctic Armageddon" (Kerr, 2010).

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